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IMITATIONAL MODELLING FORMATION OF STUDENTS' EMPIRIC KNOWLEDGE

The aim of teaching physics is not only the provision of students with the complex of theoretical knowledge connected with principles, laws, abstract objects and models but also the formation of knowledge of the system of physical experiment which is the basis of modern physics. That is to say, a pupil should not only master the theory but also know the facts proving the main theoretical aspects and forming the system of empiric knowledge of physics course. The aim of this work is the construction of qualitative and then quantitative model of the formation of the system of students' empiric knowledge which is a complex of interrelated facts obtained as a result of conducting of physical experiments.

1. The qualitative model of the formation of empiric knowledge. The systems analysis of the formation of students' factual knowledge allows to pick out the following structural elements: a student, a teacher, the environment, the system of educational experiments and observations (Fig. 1). The environment which is the complex of phenomena of everyday life influences the student and the teacher and the latter takes that into account choosing methods of teaching allowing the most optimal way of educational process.

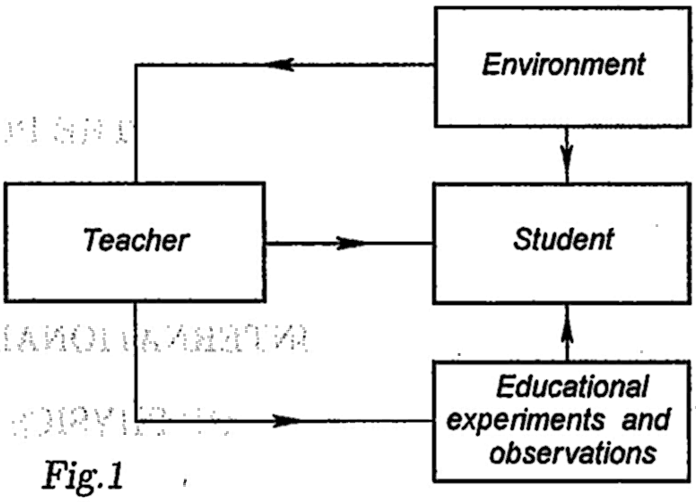

Fig.1

The analysis of the above presented model allows to pick out the following three categories of physical experiments and their corresponding facts.

1. Experiments and observations conducted by a student in his everyday life. These are the experiments proving the existence of Archimedes force, the radiation of light by a heated body, the flow of the electric current and others. It is understood that these experiments and observations can be performed in educational conditions.

2. Experiments and observations which are not performed in students' everyday life but which can be performed in educational conditions. Such are the experiments proving the facts of the existence of photoeffect, light polarization and refraction of electromagnetic waves.

2. *Experiments which can not be conducted at physics lesson and which are studied theoretically.* Such are the experimental demonstrations of facts of the existence of thermonuclear reaction, relativistic deceleration of time, X–ray diffraction and other experiments.

It is evident that in general the character of study and the speed of forgetting of empiric material corresponding to the above listed categories of physical experiments and facts are different. We may suppose that the facts of the first category are mastered quicker and forgotten slower because the student continuously confronts them "rediscovering" these facts in his everyday activity. The facts of the second category seem to be forgotten quicker than those of the first one, but not so quickly as the facts of the third category because some sensual visual image of the conducted phenomenon or object has been created by means of the educational experiment. The rate of forgetting of the facts of the third category which are studied theoretically seems to be the highest.

2. The basic items of the mathematical model. Different methods of modelling of social processes and the process of education among them are expounded in works [1–8]. The following principles make the basis of the model being used by us.

1. The information conveyed to students I, the amount of knowledge Z are representable in the form of equivalent weakly interacting elements the number of which is proportional to the number of I, Z. For the system of empiric knowledge facts or experiments proving its existence are such elements.

2. The process of teaching is a superposition of learning that is mastering the knowledge and forgetting. The rate of change of student's knowledge amount is equal to the difference of rate of mastering knowledge dZ_y/dt and the rate of forgetting dZ_3/dt :

$$\frac{dZ}{dt} = \frac{dZ_y}{dt} - \frac{dZ_3}{dt}. \tag{1}$$

3. Learning that is mastering of one element of the material takes an infinitesimal time interval as compared with the duration of study of the whole subject course. In the process of learning the student masters all the information which is conveyed to him:

$$dZ_y = dI, \qquad dZ_y/dt = dI/dt. \tag{2}$$

4. The rate of forgetting is proportional to the amount of the student's knowledge:

$$\frac{dZ_3}{dt} = \gamma Z. \tag{3}$$

3. The construction of the mathematical model. Let us subdivide the educational process into the intervals of τ duration and consider the educational material within each interval to be distributed uniformly, that is the rate of conveying information to the student remains constant: $v = dI/dt = const$. From (1)–(3) it follows that

$$\frac{dZ}{dt} = v - \gamma Z. \tag{4}$$

Supposing that at the moment of time countdown t_0 the amount of the student's knowledge $Z(t_0) = Z_0$, we get the integral:

$$\int_{Z_0}^{Z} \frac{dZ}{Z - v/\gamma} = -\gamma \int_{t_0}^{t} dt. \tag{5}$$

It follows that the amount of student's knowledge at the time moment $t = t_0 + \tau$ is equal to:

$$Z(t) = \frac{v}{\gamma}(1 - e^{-\gamma(t-t_0)}) + Z_0 e^{-\gamma(t-t_0)} = \frac{v}{\gamma}(1 - e^{-\gamma\tau}) + Z_0 e^{-\gamma\tau}. \tag{6}$$

Substituting $Z_0 = 0$, we get the amount of the mastered knowledge being equal to $(1 - e^{-\gamma\tau})v/\gamma$. If the rate of information forthcoming $v = 0$ then the amount of the nonforgotten knowledge during the τ period is equal to $Z_0 e^{-\gamma\tau}$.

Because the amount of knowledge at the end of the j-th academic year is equal to the sum of knowledge mastered in the 1st, 2nd, ..., i, ..., j-th forms and partially forgotten during $(j-1)$, $(j-2)$, ..., $(j-i)$, ..., 0 years correspondingly, we have:

$$Z_j = \sum_{i=1}^{j} \Delta Z_i e^{-\gamma(j-i)} = \sum_{i=1}^{j} \frac{v_i}{\gamma}(1 - e^{-\gamma\tau}) e^{-\gamma(j-i)}, \tag{7}$$

where $\Delta Z_i = (v_i/\gamma)(1 - e^{-\gamma\tau})$ is the knowledge obtained in the i-th form.

The use of this model for the research of empiric knowledge system formation requires taking account of the dependence of forgetting time on the category of facts. Considering the forgetting coefficients of facts of the first, second and third categories to be equal to $\gamma_1, \gamma_2, \gamma_3$; and their forthcoming rates v_{i1}, v_{i2}, v_{i3}, where $i = 1, 2, \ldots, 11$ is the form number, we get:

$$Z_j = \sum_{k=1}^{3} Z_{jk} = \sum_{k=1}^{3} \sum_{i=1}^{j} \frac{v_{ik}}{\gamma_k}(1 - e^{-\gamma_k\tau}) e^{-\gamma_k(j-i)}, \tag{8}$$

where Z_{jk} is the amount of the student's knowledge corresponding to the facts of the k–th category at the end of the j–th academic year. By the coefficient of formation completeness of empiric knowledge we will imply the ratio of the students knowledge to the information which was conveyed to them: $K' = Z/I$.

4. Matching of the model with testing results. As a result of the analysis of standard nature study, natural geography and physics textbooks the values have been defined of the empiric knowledge forthcoming rates in different forms on different physics branches for facts of the 1st, 2nd and 3rd categories in the units of fact/year. Only those facts were taken into account which are provided with drawn figures in the textbook: it was supposed that the amount of empiric knowledge in the textbook is proportional to their number.

A program has been developed in *Pascal* language for imitational modelling of the process of the formation of the system of empiric knowledge allowing with the known forgetting coefficients γ_k and values v_{ik}, where $k = 1, 2, 3$, and $i = 1, 2, \ldots 11$ to calculate the level of an average statistical student's empirical knowledge at a given time moment on the whole physics course, its different chapters and different categories of facts.

It will be understood that the results of such a research are determined by v_{ik} values obtained from the analysis of school textbooks [9–13] and forgetting coefficients γ_k of facts of different categories. To approximately evaluate the coefficient of forgetting the testing has been conducted of 40 1st year students of Glazov Teachers' Training Institute which involved the determination of the level of knowledge of 50 educational facts (10 from each physics branch). It allowed to evaluate the coefficient of empiric knowledge formation completeness on different categories of facts as the ratio of the number of asked questions N to the number of correct answers $n : K = n/N$.

The task of matching of the mathematical model with the results of testing implies the determination of values $\gamma_1, \gamma_2, \gamma_3$ for which the coefficients of formation completeness of empiric knowledge K'_k for facts of different categories k predicted by the model would be the closest to K_k values obtained from the testing. For this purpose we have used the method of the least squares involving the minimization of the sum of squares of differences:

$$S = \sum_{k=1}^{3} (K_k - K'_k)^2 = min. \quad (9)$$

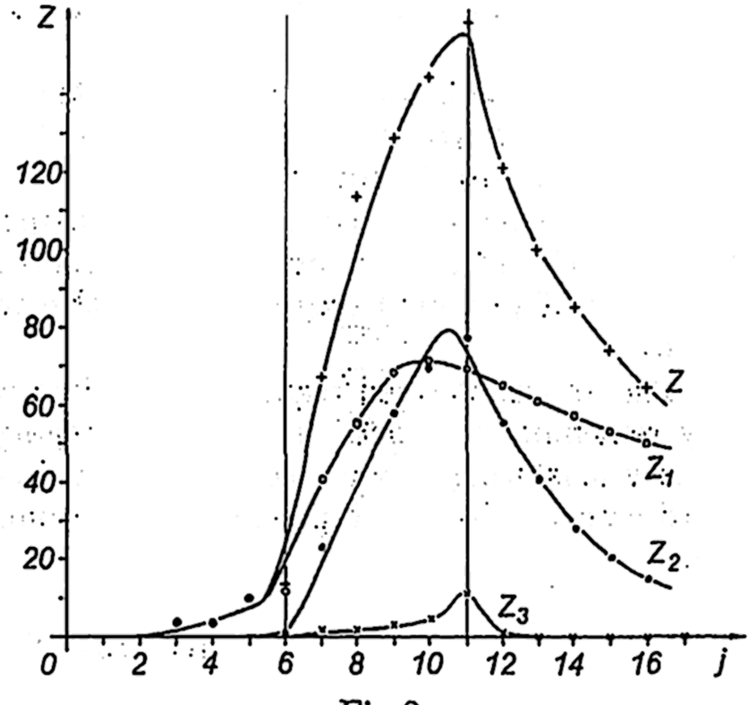

Fig.2

A program has been made for optimization of parameters $\gamma_1, \gamma_2, \gamma_3$ using the method of coordinate–wise descent in a few iterations minimizing the sum S. As a result of its run the following values have been obtained: $\gamma_1 \approx 0,06$, $\gamma_2 \approx 0,3$ and $\gamma_3 \approx 2$.

Thus, physical experiments and facts of the third category studied at the purely theoretical level are forgotten most quickly. The period of forgetting a half of information is $T_3 = \ln 2/\gamma_3 \approx 0,3$ years. The facts studied with the provision of physical experiment are forgotten somewhat slower, the corresponding period of forgetting a half of information is equal to $T_2 = \ln 2/\gamma_2 \approx 2$ years. And the facts of the first category which can be experimentally proven by the student in his everyday life are forgotten most slowly: $T_1 = \ln 2/\gamma_1 \approx 11$ years. It should be remembered that the indicated values γ_k, T_k are approximate because they were obtained as a result of testing of a relatively small number of school leavers.

The obtained values $\gamma_1, \gamma_2, \gamma_3$ allow to draw the graphs of dependence of the amount of empiric knowledge on the time. Figure 2 illustrates the graphs of

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