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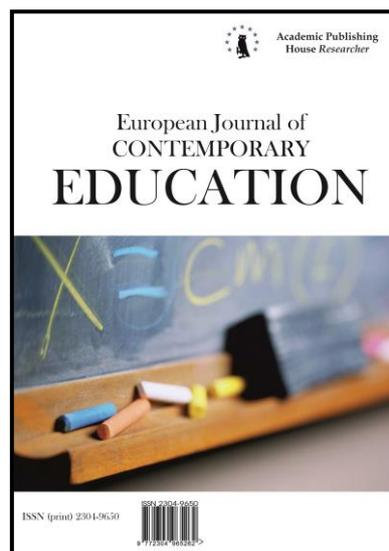
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Computer Model of the Empirical Knowledge of Physics Formation: Coordination with Testing Results

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Abstract

The use of method of imitational modeling to study forming the empirical knowledge in pupil's consciousness is discussed. The offered model is based on division of the physical facts into three categories: 1) the facts established in everyday life; 2) the facts, which the pupil can experimentally establish at a physics lesson; 3) the facts which are studied only on the theoretical level (speculative or ideally). The determination of the forgetting coefficients of the facts of the first, second and third categories and coordination of imitating model with distribution of empirical information in the school physics course and testing results is carried out. The graphs of dependence of empirical knowledge for various physics sections and facts categories on time are given.

Keywords: didactics, imitation models, mathematical methods, pedagogics, pedagogical examination, computer simulations, theory of training.

1. Introduction

At the junction of didactics and mathematics the mathematical theory of training appeared, which provides research of the system "teacher – pupil" by methods of mathematical modeling (Dobrynina, 2009; Roberts, 1986; Hunt, 2007). Development of information technologies has led to the emergence of imitating models of didactic systems (Dorrer, Ivanilova, 2007; Kudrjavcev et al., 1996). The essence of the modeling method is that the real pedagogical system is replaced with an abstract model, – some idealized object which behaves like the studied system. Such model can be a system of logical rules, mathematical equations or a computer program allowing to make a

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series of computational experiments at various parameters, entry conditions and external influences. Changing an initial condition of the pupil, distribution of educational information and other parameters of the modelled didactic process, it is possible to study their influence on the result of training (Atkinson et al., 1969).

Now the discrete and continuous computer models of training which are based on automatic approach and on the solution of the differential equations (Leont'ev, Gohman, 1984; Mayer, 2013; Novikov, 1998) are known. Sometimes the multi-agent models, by which each pupil is replaced with a program agent functioning irrespective of other agents, are used (Ivashkin, Nazojkin, 2011). Also there are imitating models using Petri's networks, genetic algorithms, matrix modeling and others (Solovov, Men'shikov, 2001; Firstov, 2011). In all these cases there is a problem of the computer model coordination with the distribution of educational information and results of testing. It is necessary to set certain numerical values to the pupil's parameters, the speed of information transfer, the level of teacher's requirements during training; only in this case the model will describe a real situation.

The purpose of the physics course is to construct a physical picture of the world in the pupil's consciousness. Because of bifurcation of the world into the external (open) and internal (hidden) parties, in the theory of knowledge there is a phenomenon (external aspect) and essence (internal aspect). The outer side of the object (a phenomenon) is perceived by the person's sense organs directly or by means of devices. This leads to emergence of the empirical or factual knowledge as the result of observation and experiments. The generalized facts and empirical laws are elements of empirical knowledge. Comprehension of the inside of researched object (that is its essence) leads to receiving theoretical knowledge.

The empirical knowledge is an important component of the physics course, therefore the formation of the empirical knowledge system in the pupil's consciousness is an actual problem. To study it, methods of mathematical and computer modeling can be used. The purpose of the present work is as follows: 1) creation of the imitational model of the empirical knowledge assimilation at physics lessons at Russian schools; 2) coordination of the model with distribution of educational information in a school course of physics and the testing results of the school graduates.

2. Division of facts into three categories

The studying method of this or that fact depends on the possibility of its experimental establishment in everyday life and at school. It is obvious that almost all facts established in everyday life can be studied experimentally at the physics lesson. Therefore we have formulated three following categories of the facts differing in the way of their studying by pupils (Mayer, 2014):

1. The facts of the first category which can be established by an average pupil in the everyday life after experimental studying them at school. Such facts belong to this category as: the existence of Archimedes force, heat exchange, electrization by friction, light reflection and refraction, heating of the conductor at course of current, a luminescence of heated bodies, thermal expansion of bodies and others. Certainly, these facts can be also experimentally established while training.

2. The facts of the second category can't be experimentally established by pupils in everyday life, however they can be experimentally proved at the physics lessons: Kirchhoff's laws, electromagnetic induction, electric resonance, photo-effect, polarization of light, refraction of electromagnetic waves.

3. The facts of the third category can't be experimentally established in the conditions of training, and their studying is carried out on the speculative level by means of educational video movies and computer models. For example, the facts of existence of light pressure, thermonuclear reaction, relativistic delay of time, results of Michelson's experiment.

It is obvious that assimilation durability and the forgetting speed for the facts of first, second and third categories are various. It is possible to assume that the facts of the first category are easier and strongly acquired and forgotten slower as they are included in activity of the pupil who continuously faces them, "rediscovering". The facts of the second category are forgotten quicker, than the first, but not so quickly, as the third category facts because the pupils have observed or performed the corresponding experiments at physics lessons. The forgetting speed of the third category facts is the highest, because the pupils studied them speculative (or ideally) and seldom use this knowledge in their everyday activity. Because of a big variety of the facts and their features

the offered division into three categories is not absolute and has no clear boundary: there are facts which can be taken simultaneously to the first and the second or to the second and the third categories. However, in most cases it is possible to tell definitely, what category this or that fact belongs to.

3. Mathematical model of assimilation of empirical knowledge

Let us break training process into small intervals and consider that in each interval the speed of information transfer to the pupil is constant: $v = dI/dt = const$. Let us consider, that all educational information is acquired by the pupil. The speed of increase in the pupil's knowledge Z is equal to the sum of the speed of training and speed of forgetting $-\gamma \cdot Z$: $dZ/dt = v - \gamma \cdot Z$. Here γ is the forgetting coefficient. Considering that at the beginning of training t_0 the amount of the pupil's knowledge is $Z(t_0) = Z_0$, we get:

$$\int_{Z_0}^Z \frac{dZ}{Z - v/\gamma} = -\gamma \int_{t_0}^t dt.$$

It follows that amount of the pupil's knowledge in a timepoint t is equal to:

$$Z(t) = \frac{v}{\gamma}(1 - e^{-\gamma(t-t_0)}) + Z_0 e^{-\gamma(t-t_0)}.$$

Let at an initial timepoint $t_0 = 0$ the amount of knowledge of the pupil to be equal to Z_i . The amount of knowledge of the pupil at the end of the $(i + 1)$ – the academic year:

$$Z_{i+1} = Z_i e^{-\gamma\tau} + \frac{v_{i+1}}{\gamma}(1 - e^{-\gamma\tau}),$$

where Z_i – the level of knowledge at the end of i –th of year, v_{i+1} – the speed of the knowledge transfer in the $(i + 1)$ –th year and $\tau = 1$ year. This equation allows to calculate sequentially the amount of the pupil's empirical knowledge at the end of the 1st, 2nd, ..., 11th years of training at school.

As amount of the pupil's facts knowledge of the j –th academic year is equal to the sum of the knowledge acquired in the 1st, 2nd, ..., i –th, ..., j –th classes and partially forgotten during $(j - 1)$, $(j - 2)$, ..., 1, 0 years respectively, we have:

$$Z_j = \sum_{i=1}^j \Delta Z_i e^{-\gamma(j-i)\tau} = \sum_{i=1}^j \frac{v_i}{\gamma}(1 - e^{-\gamma\tau}) e^{-\gamma(j-i)\tau},$$

where $\Delta Z_i = (v_i / \gamma)(1 - e^{-\gamma\tau})$ – the quantity of knowledge acquired in the i –th class, the multiplier $e^{-\gamma(j-i)\tau}$ considers the forgetting within $(j - i)$ years, $\tau = 1$ year – training time in one class.

Using this model for research of knowledge assimilation demands to account the dependence of the forgetting time on the category of the facts. Considering that forgetting coefficients of the facts for the first, second and third categories and their transfer speeds are respectively equal to γ_1 , γ_2 , γ_3 and v_{i1} , v_{i2} , v_{i3} , where $i = 1, 2, \dots, 11$ – number of a class, after transformations, we receive:

$$Z_j = \sum_{k=1}^3 Z_{jk} = \sum_{k=1}^3 \sum_{i=1}^j \frac{v_{ik}}{\gamma_k}(1 - \exp(-\gamma_k\tau)) \exp(-\gamma_k(j-i)\tau),$$

where Z_{ik} – the amount of the pupil’s knowledge corresponding to the facts of k –th category at the end of the j –th class. We offer an assimilation coefficient of empirical knowledge K_j as the relation of the factual knowledge Z_j in the j –th class to the total empirical information: $K'_j = Z_j / I_j$. Thus, the quantity of the given information is equal: $I_j = \sum_{k=1}^3 \sum_{i=1}^j \nu_{ik} \tau$.

Table 1. Quantity of the facts of first, second and third categories in various sections of the school physics course

Class i	Category of facts	Mechanics	Theory of heat	Electrodynamics	Optics	Quantum physics
7	1	32	13	0	0	0
	2	13	0	0	0	0
	3	1	0	0	0	0
8	1	1	13	12	9	0
	2	0	13	32	4	0
	3	0	0	3	0	0
9	1	27	0	0	0	0
	2	23	0	0	0	0
	3	1	0	0	0	0
10	1	2	9	16	0	0
	2	2	13	42	0	0
	3	0	3	8	0	0
11	1	2	0	2	3	2
	2	2	0	28	13	13
	3	1	0	8	6	37

4. Coordination of model with results of testing

As a result of the content analysis of the Russian physics textbooks the transfer speed of empirical knowledge in different classes in fact/year have been determined. This allows take into account the division of the facts in dependence on the sections of physics and on the categories (Table 1). To determine the forgetting coefficients, the testing of about 100 first–year students of the Physics Department of the Glazov State Pedagogical Institute has been held. It allows to determine approximately the these student’s knowledge level of 50 educational facts (10 from each section of physics) and estimate the assimilation coefficients K of the facts of the first, second and third categories as the relation of number N of the asked questions to the number n of the correct answers: $K = n / N$. The received results are presented in the second column of Table 2.

The problem of coordination of mathematical model with the testing results is reduced to determination of such values $\gamma_1, \gamma_2, \gamma_3$ at which the assimilation coefficients of empirical knowledge K'_k for the facts of various categories predicted by the model, are closest to the

corresponding values K_k received at testing. For this purpose we use the method of the smallest squares consisting in minimization of the sum:

$$S = \sum_{k=1}^3 (K_k - K'_k)^2 = \min .$$

For the optimization of the parameters $\gamma_1, \gamma_2, \gamma_3$ and modeling of empirical knowledge learning, the computer programs and files have been created:

1. A data file which consists of the information about distribution of the facts in the school physics course and about test result of the school graduates in a half year after school ($t = 11,5$ years). From the test results we get that the pupil's knowledge levels for the facts of the first, second and third categories are respectively equal to $K_1 \approx 0,72, K_2 \approx 0,35, K_3 \approx 0,19$.

2. The subprogramme which allows, using the information about facts distribution in the physics course and coefficients $\gamma_1, \gamma_2, \gamma_3$, to calculate the theoretical (predicted by the model) values of the empirical knowledge levels for facts of various categories K'_k in timepoint $t = 11,5$ years.

3. The program for determination of values γ_k ($k = 1, 2, 3$) at which the sum of squares of differences K_k and K'_k is minimum. It adjusts of the model to the results of testing and distribution of the empirical information.

4. The program which carries out imitating modeling of learning of empirical knowledge for the facts of the first, second and third categories and various sections of physics.

Table 2. Coordination of model with results of testing

		Result of testing	MODEL			
			1	2	3	4
Level of the facts knowledge	K_1	0,72	0,30	0,50	0,73	0,73
	K_2	0,35	0,43	0,62	0,17	0,35
	K_3	0,19	0,60	0,19	0,32	0,19
The forgetting coefficients	γ_1	—	0,38	0,20	0,090	0,090
	γ_2	—	0,38	0,20	0,95	0,49
	γ_3	—	0,38	1,48	0,95	1,5
Criterion	S	—	0,36	0,12	0,049	0,00016

5. Determination of forgetting coefficients

To prove the necessity of division of the facts into three categories, we have carried out calculations for four cases: 1) the facts of all three categories have identical coefficients of forgetting: $\gamma_1 = \gamma_2 = \gamma_3$; 2) the facts of the first and the second categories have the identical forgetting coefficients $\gamma_1 = \gamma_2$ which are not equal to γ_3 ; 3) the facts of the second and third categories have the identical forgetting coefficients $\gamma_2 = \gamma_3$ which are not equal to γ_1 ; 4) the forgetting coefficients of the facts $\gamma_1, \gamma_2, \gamma_3$ of the first, second and third categories are different. The criterion of proximity of the results given by the model to the testing results is the sum S

which at their coincidence is equal to zero. Values S for these four models are presented in the third, fourth, fifth and sixth columns of tab. 2 respectively.

From the received results it is visible that the fourth model considering the distinction of the forgetting coefficients of the facts of the first, second and third categories most precisely corresponds to the results of testing. So, the required forgetting coefficients are approximately equal to $\gamma_1 \approx 0,090$, $\gamma_2 \approx 0,49$, $\gamma_3 \approx 1,5$ (1/year).

Limits of applicability of the offered model are defined by an error of the data entered into the model, and by influence of some unaccounted and uncontrollable factors. For example, when modelling it was supposed that studying of physics was carried out without breaks and the speed of receipt of educational information within a year was constant; the training of pupils for final and entrance examinations wasn't considered. The accounting of these factors demands essential complication of the model, introduction of new variables, assessment of which would require more complex testing. In the present work the first approach which allows to estimate the parameters γ_1 , γ_2 , γ_3 and to determine the main features of process of empirical knowledge formation is made.

6. The result of modelling of the empirical knowledge assimilation

The curves of dependence of the pupil's empirical knowledge from time are presented on fig. 1. It is visible that the facts of the third category studied at purely speculative level are most quickly forgotten. Their forgetting coefficient is $\gamma_3 \approx 1,5$ 1/year, the period of the half information forgetting is $T_3 = \ln 2 / \gamma_3 \approx 0,46$ years. The facts studied with the help of physical experiments are forgotten slightly slower: $\gamma_2 \approx 0,49$ 1/year, $T_2 = \ln 2 / \gamma_2 \approx 1,4$ year. The first category facts, which the pupil can establish experimentally in everyday life are forgotten even slower: $\gamma_1 \approx 0,090$ 1/year, $T_1 = \ln 2 / \gamma_1 \approx 7,7$ 1/year. The received values γ_1 , γ_2 , γ_3 allow to calculate and construct graphs of dependences of the empirical knowledge quantities on time for various categories.

In fig. 1 general changes of the pupil's empirical knowledge with the time, and also the knowledge of the facts of the first, second and third categories are shown. In fig. 2.1 and 2.2 it is shown how the amount of empirical knowledge of the physics sections changes: for mechanics (curve 1), for molecular physics and thermodynamics (curve 2), for electrodynamics (curve 3), for optics (curve 4), for quantum physics (curve 5). Failures in graphs are connected with forgetting in intervals between the first and second studying of the given section of physics, taking place at the first (7–9 classes) and second step (10–11 classes) of training. It is visible that empirical knowledge of various sections of physics is forgotten with a different speed. For example, the level of formation of the empirical knowledge of mechanics consisting mainly of the first facts category decreases slowly (fig. 2.1, a curve 1) while the facts of quantum physics which are generally relating to the third category are forgotten quicker (fig. 2.2, a curve 5).

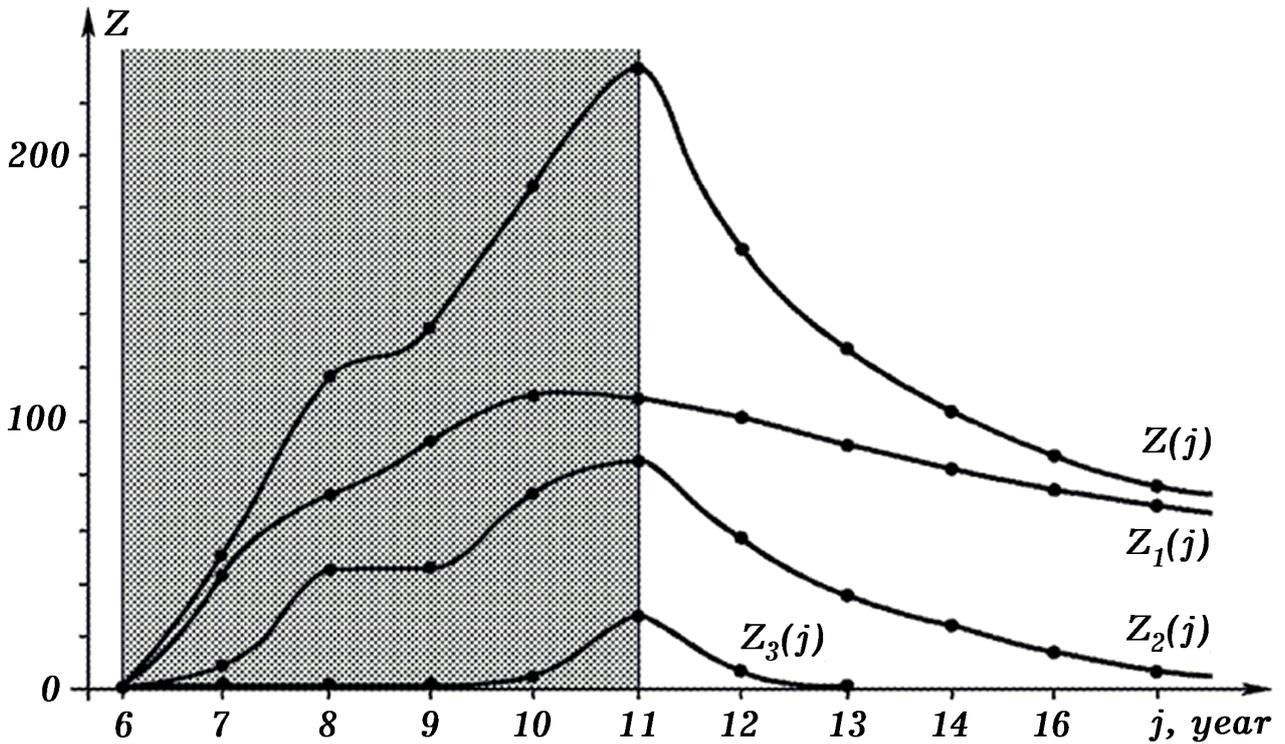


Figure 1. Dependence of factual knowledge of various categories on time

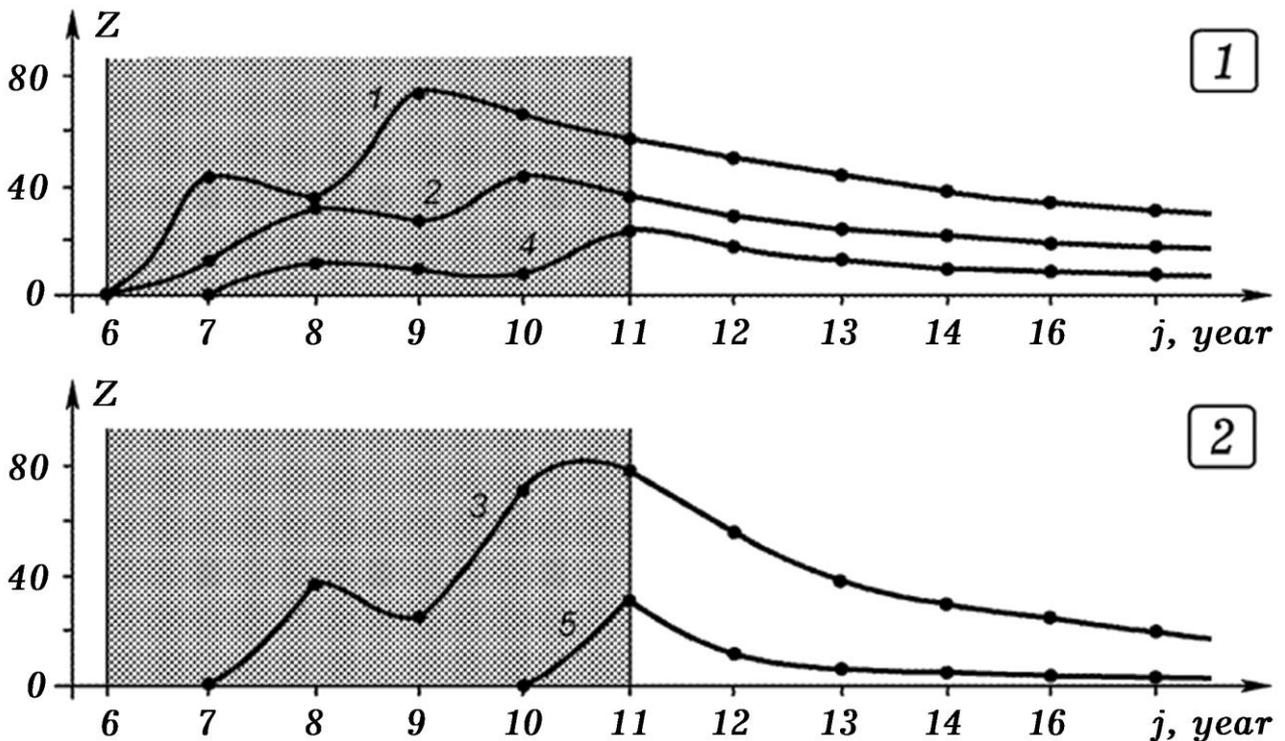


Figure 2. Dependence of factual knowledge of various physics sections on time

It follows from Figure 1 and Figure 2 that the level of knowledge of the first category facts smoothly increases to some value, and then remains almost invariable. The knowledge levels of the second and third category facts smoothly increase, at the end of training they reach a maximum, and then exponential decrease. Correspondence of modeling results to pedagogical experience confirms the validity of initial assumptions. First of all, it is about a hypothesis of expediency to

divide the facts into three categories that allows to consider their dissimilarity from the didactic point of view caused by occurrence of some facts in everyday life and possibility of their experimental establishment at a lesson.

The analysis of the received graphs and testing results allows to formulate the following regularities of the empirical knowledge formation in the pupil's consciousness:

1. In studying process the knowledge level of the first category facts entering the pupil's daily experience increases, and after training remains almost unchanged.

2. After studying the levels of knowledge of the second and third category facts which do not enter pupil's daily activity decreases due to forgetting.

3. The more the studying of the second and third category facts is based on the pupil's activities, connected with observations and performance of educational experiments or their theoretical (speculative) studying, the less is the forgetting speed of those facts.

7. Conclusion

The article presents application of mathematical and computer models for studying of the empirical knowledge assimilation by pupils. The three-component model which bases on division of the physical facts into three categories is offered. For approximate assessment of the forgetting coefficients of the first, second and third categories facts the coordination of imitating model with distribution of empirical information in a school physics course and results of testing is carried out. The graphs of dependence of empirical knowledge for various physics sections and categories of facts on time are received. The above described regularities explain why school graduates usually remember the facts of the first category well enough, but satisfactory – the facts of the second category, and rather poor – the facts of the third category. Further specification of the computer model of the empirical knowledge assimilation by pupils through the use of more accurate data about distribution of empirical information and testing results is possible. The above mentioned approach can be used to simulate the process of learning in other academic disciplines.

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