THE COMPLEXITY ASSESSMENT OF CONCEPTIONS AND EDUCATIONAL TEXTS ON NATURAL SCIENTIFIC DISCIPLINES

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Abstract

Improving the teaching needs to determine the characteristics of various elements of learning material (ELM). The article considers the problem of measuring didactic complexity of the conceptions, statements and texts, which are used at school. The methodological basis of the research are the works by V. S. Avanesov, B. Bitinas's, G.V.Glass, J.C.Stanley, V.P. Bespalko, Ya.A. Mikk, I.V. Oborneva.

The didactic complexity of ELM is offered to be understood as the characteristic which is proportional to the time needed to assimilate the information presented in this ELM. Uniform criteria for estimation of the scientific conceptions complexity on various natural science subjects are developed and the method of the complexity assessment of the statements (laws, principles, postulates) based on abstraction scale is offered. For an assessment of ELM complexity it is necessary to provide the teacher who, explaining the essence of statement, describes the corresponding situation and gives the definitions to all terms used.

The offered method consists in the following: 1) to create the text including the formulation of the law, the description of the situation (the phenomenon, the experiment, etc.) corresponding to this law and the definitions of the science terms used; 2) to compile a dictionary–thesaurus and to estimate the didactic complexity of the conceptions entering it; 3) to start a special computer program which analyzes the text and determines the conceptions quantity and their total complexity.

In case of large texts assessment it should be taken into account that the didactic complexity of the issues studied at school is determined by a variety and abstractness of the qualitative and quantitative models used. The developed estimation method of didactic complexity of the physics textbooks includes: 1) the determination of the physics complexity of the textbook summing up the complexity of the physical objects, phenomena, experiments, statements and theories; 2) the measurement of mathematical complexity of the textbook by counting the number of formulas (considering their complexity) and drawings presenting mathematical abstractions (a vector, power lines, graphs); 3) the calculation of the total index of the textbook didactic difficulty.

With the help of this method 16 Russian physics textbooks for school and university have been analyzed, their distribution within the characteristics space "physical complexity - mathematical complexity" has been studied.

For the determination of the didactic complexity of the textbooks on natural science disciplines the method of pair comparisons of various texts is used. The textbooks are compared according to the following characteristics: 1) the variety and abstractness of qualitative explanations, their isolation from everyday life; 2) the complexity of mathematical methods and models, the variety of solved quantitative tasks.

The Russian textbooks on nature study, geography, biology, physics, chemistry (21 textbooks) have been analysed, and their paired comparison with each other has been made. It is found that the chosen characteristics can be considered as independent; the correlation coefficient between them is equal 0.51. On the basis of the obtained data the distribution of school textbooks on physics, chemistry, biology, ecology within space of characteristics "complexity of qualitative models – complexity of quantitative models" is presented, their classification is made.

Keywords: conceptions, didactics, education, mathematical methods, nature sciences, textbook, training, qualimetry.
1 INTRODUCTION

In many respects the level of assimilation by the schoolchildren of natural science subjects depends on the quality of teaching materials used, their complexity and comprehensibility for the pupil. The training material stated in textbooks and its complexity should correspond to the modern content of science and psychological characteristics of pupils, their capabilities to acquire and comprehend the obtained knowledge. Creation of new textbooks and techniques of teaching physics, chemistry, biology and other subjects requires development of methods to measure didactic characteristics of various elements of a training material (ELMs), such as theory pieces, practical and theoretical tasks, descriptions of scientific experiments, etc. The problem of the content analysis of educational texts and their complexity estimation is urgent [1-4].

Teachers and school students, comparing various subject matters, intuitively "react" to the level of abstractness of the studied questions, therefore, as V.P. Bespalko notes, criterion of complexity of the educational text first of all is its theoretical nature and abstractness degree [2, p. 97–98]. It depends on a ratio between the pupil's experience and the content of a training material: "the subject seems to the pupil the more difficult, the more different the levels of the textbook abstraction and previous experience of the pupil are" [2, p. 98]. The greater complexity and lower the level of knowledge, the more difficult the training material for the student and the less its availability are. Ya.A. Mikk claims that "scientific character of material without availability is senselessly: there is no sense to train if pupils can not acquire the training material" [4, p. 3].

Various approaches to the complexity measurement of the educational text are possible [4–6]. For example, Ya.A. Mikk's book [4, p. 32] allocates the following components of the text complexity: 1) information content; 2) linguistic complexity; 3) structure clarity; 4) abstractness of narration. It means that the linguistic complexity of the text depends on a variety of the vocabulary, average length of words and average length of sentences. To determine the abstractness level of the text scientists use the method of content analysis [5–9], apply the concreteness–abstractness scales, or count the number of words with abstract suffixes [4, p. 45].

Any text consists of judgments, each of them is not just a collection of words, but a system of the interconnected concepts; therefore the text complexity, strictly speaking, is not equal to the sum of the complexities entering concepts. From the systems theory [10] it is known that complexity of any system depends on the degree of the variety, quantity and complexity of the elements (subsystems) and links between them. The statement consisting of "simple" concepts can express a complex idea which can be difficult to understand for many pupils. It is possible to consider the "semantic" component of the text complexity if to perform an expert estimation of all text, without dividing it into separate terms. At the same time the expert can: 1) estimate didactic complexity (DC) of the educational text according to some scale, receiving numerical values; 2) use a method of pair comparisons, comparing each two educational texts from a given set and calculating DC for each text.

In this article the problem of an approximate assessment of the didactic complexity (DC) of educational texts on physics and other natural-science subjects is considered. The methodological basis of the research are the works by V. S. Avanesov, B. Bilinas's, G.V.Glass, J.C.Stanley, V.P. Bespalko, Ya.A. Mikk, I.V. Oborneva. Two methods of the DC determination of educational texts are used: 1) by counting the scientific terms number and taking into account their complexity with help of the computer; 2) by the method of pair comparisons of various texts with each other.

2 ESTIMATION OF DIDACTIC COMPLEXITY OF THE TEXT BY COUNTING CONCEPTS AND CONSIDERING THEIR ABSTRACTNESS

In our opinion, didactic complexity of the text must be reduced to values that are measurable and have practical implications for the organization of training. Such values are: 1) the number of words which are pronounced by the teacher or read by the pupil; 2) the time required for perception or assimilation of the given information block. It is possible to imagine an average pupil who has successfully finished the 5th class (the 1st year in the Russian secondary school), to whom the teacher explains the essence of the matter stated in the text, reporting all necessary definitions of the used concepts and the formulations of laws. The more words the teacher pronounces, the more time it takes and the higher complexity of the text is. If it turns out that for studying the biological cell structure it is necessary to spend 1 hour, and mastering integration needs 10 hours, so it follows that the DC of concept "integral" is 10 times more than the DC of the concept "cell".

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For most of pupils the main difficulty is not in the actual reading of the text, but understanding and assimilation of some rather abstract ideas about the Universe structure, processes happening in a living organism, the atoms and molecules structure, etc., as well as acquiring skills to apply that knowledge to solve corresponding tasks. If to assume that the pupil does not have difficulties with reading long words and sentences, then it is possible to ignore linguistic complexity. In this case the didactic complexity (DC) of the educational text is determined only by its content, or sense. We can assume that the DC is approximately equal to its level of abstraction, which is proportional to the number and complexity of the concepts, judgments, logical conclusions, mathematical expressions and drawings with abstract information. The educational text containing the concepts "sine", "logarithm", "derivative", "integral" is notably more difficult than the text about diffusion or the Solar eclipse explained at a qualitative level.

One of approaches to determine the educational text DC consists in counting number of the concepts entering it and considering their complexity. If the text contains a picture or a formula (mathematical or chemical), then it should be replaced with the shortest verbal description, and after that we can determine the text complexity [11]. It is supposed that DC of the text is proportional to the sum of the complexities of all words entering the given text.

It is necessary to develop criteria for assessing the complexity of the words (concepts) in an educational text. If the word (proper or common) is included in the dictionary in physics, mathematics, chemistry, biology, etc., then it is a scientific term. Let us imagine that the text under analysis is read by an average schoolchild, who has successfully completed the 5th class (1st year of secondary school), and who is further referred to as "the pupil". The didactic complexity (DC) of the words entering the text is determined as:

DC $s_1 = 1$: the words which are not scientific terms and used by pupils in everyday life ("falls", "flows", "rotates").

DC $s_2 = 2$: 1) the scientific terms with a low degree of abstraction, studied in 1–5 classes, used by the school student in everyday life and do not require any explanations ("sphere", "air", "evaporation", "soil", "plant", "Moon"); 2) the proper names which are often used in everyday life, indicating geographical, astronomical and other objects that are shown on television and do not require special explanations; 3) the arithmetic operations: addition, subtraction, multiplication, division.

DC $s_3 = 3$: 1) the scientific terms, having an average degree of abstraction that are not used in everyday life and requiring simple explanations; they designate or describe phenomena and objects, with the likes of which the pupil meets daily ("impulse", "voltage", "cadmium") or which he can observe very seldom ("Solar eclipse", "plant cell", "ruby"); 2) the proper names designating geographical, astronomical and other objects which are seldom used in everyday life and badly known to the pupil and requiring explanations ("tropic", "Pluto"); 3) mathematical terms: a linear relationship, direct or inverse proportionality, the vector sum or difference.

DC $s_4 = 4$: 1) the scientific terms of high abstraction, corresponding events and objects that the pupil can not in principle feel with the sense organs and should force to strain his imagination to analyse them. They are not similar to the objects observed in everyday life ("double star", "neutron", "gene", "zygote", "kernel of cell", "molecule" without taking into account the structure, "mole"). This category includes the micro- and mega-world objects, as well as abstract concepts that are difficult to explain to the student. 2) mathematical operations and functions: raising to a noninteger power, root extraction, sine, cosine, tangent.

DC $s_5 = 5$: 1) the scientific terms of very high abstraction, which are denoting objects and processes, consisting of a large number of components (particles) that the pupil can not observe in principle ("sodium atom" as a system of 11 protons, 12 neutrons and 11 electrons, "DNA molecule" with the structure); 2) the mathematical terms designating difficult functions and operations: potentiation, logarithms, differentiation, integration, finding of a limit, the scalar product of vectors. This group includes the terms which are very difficult to explain to the pupil.

Let's say it is necessary to estimate the didactic complexity of any statements, for example, Ohm's law. Imagine a physics teacher who, explaining this law, describes the corresponding physical situation (a phenomenon experiment, etc.) and gives definitions for all concepts (terms). DC of law is proportional to the spent time, so for its determination we must calculate the complexity of the text block, consisting of: 1) the formulation of the law; 2) the description of the physical situation (the phenomenon or experiments) corresponding to the given law; 3) the definition of all scientific terms.
entering the formulation of the law. And the measure unit of the text complexity is one mentioning of
the word with DC $s = 1$. The example of such text block is stated below.

**Ohm's Law for a subcircuit:** Current intensity (amperage) is directly proportional to the voltage of a
circuit section. **Analysed phenomenon:** the voltage source is connected to the resistor and an
ammeter in series, parallel to the resistor we turn on the voltmeter; by changing the voltage on the
source poles, we measure amperage. **Current intensity:** the charge passing through a conductor
during a unit of time. **Electrical charge:** the ebonite stick in case of friction against fur acquires a
charge. **Voltage:** potential difference. **Resistor:** the conductor with the known resistance. **Direct
proportionality:** the more times the argument increases, the more times the function increases.
**Resistance:** the ratio of voltage to current intensity. (Complexity of the concepts highlighted in bold in
the italic type wasn't considered; complexity of the text is 116).

The approach considered above looks like A.M. Sokhor's method [4, p. 34–35] who, for an estimation
of the text complexity divides concepts into: 1) familiar or everyday concepts which are not explained
anywhere in the text; 2) unfamiliar or scientific concepts for which the definitions are given in the text.
Analysing the text, the expert must write out definitions of all scientific concepts; in this definitions he
finds other scientific concepts, defines them, and so on until only familiar concepts remain. We can
consider that the text complexity or information depth is proportional to the number of the written-out
definitions.

To increase the objectivity of the text DC assessment and repeatability of the turning-out results it is
convenient to realize counting of terms by means of a special computer program [6, 9, 12]. This
program, using the dictionary–thesaurus, counts frequencies of mentioning of different terms in the
text file and considers their complexity DC which is defined according to the criteria given above. In
this case the expert's operations consist of the following stages: 1) the compilation of the dictionary-
thesaurus; 2) the estimation of didactic complexity of the terms entering it; 3) the preparation of the file
with the text under analysis; 4) the creation and start of the program that analyses the text; 5) the
interpretation of the results. The text block considered above has complexity 116 (in Russian). The
proposed method is useful for the DC assessment of small educational texts for which the dictionary–
thesaurus is not very large. For assessment of textbooks DC it is necessary to involve the experts,
who use the appropriate scale and criteria of complexity, the method of paired comparisons, etc.

## 3 ASSESSMENT OF THE PHYSICS TEXTBOOKS COMPLEXITY

The analysis of the full text of the textbook on the computer is a rather labor-consuming procedure, as
for this purpose it is necessary to make the extensive dictionary thesaurus. Instead of this, it is
possible to make a representative selection of pages, and with the expert help to estimate their
average complexity. If the volume of the selection is sufficiently large (for example, 30–40 pages from
400), then the result of such assessment can be extended to the whole textbook. Let us distinguish
the physical $F$ and mathematical $M$ complexities of the textbook [13]. To assess physical
complexity, it is necessary to find the degree of abstractness of the used models and the level of their
isolation from the pupil’s everyday experience. At the same time it is necessary to consider: 1) the
perception of the object with sense organs; 2) the changes of the object over time; 3) the number of
the freedom degrees; 4) the space-time length of the object or process; 5) the existence of the object
structure; 6) the existence of an apparent contradiction between of behavior of the object and
"common sense". The mathematical complexity of the text depends on variety, quantity and
complexity of the used formulas and figures containing mathematical abstractions.

Under the physical complexity of textbook let’s take the value $F$, that is equal to the sum of the
complexity of the considered physical objects, phenomena, experiments and physical theories
(postulates, ideas, consequences). The physical complexity of the textbook is estimated as follows:

1. To analyze the contents of the $i$-th textbook, if necessary, to view separate chapters, and to
assess the general complexity $A_i$ of the studied objects, the phenomena, and also physical
theories according to a scale 1-2-3-4-5: $A_i = 1$, if the physical objects and phenomena
considered in the textbook can be perceived by sense organs of the person (water, a spring, a
stopwatch, reflection of light), and their explanations are obvious and don't require imagination;
$A_i = 3$, if objects and the phenomena which are discussed in the textbook can be observed in a
physical laboratory (an oscillograph, photoeffect, electrolysis) or there are explanations for
understanding of which the pupil must imagine the molecules, atoms, gravitational and
electromagnetic fields; $A_i = 5$, if the experiments considered in textbook are non-reproducible in

$$\text{Voltage:} \quad V = \frac{E}{R}, \quad \text{Current intensity:} \quad I = \frac{E}{R}, \quad \text{Resistance:} \quad R = \frac{E}{I}$$
the conditions of training (nuclear reaction, the accelerator of elementary particles) and/or there are arguments that are contrary to "common sense" (wave-corpuscle dualism, relativity of simultaneity). The values of $A_j = 2$ and $4$ are the intermediate.

2 To select $n = 10-15$ pages of the textbook which are uniformly distributed throughout the text (for example, if in the textbook of 280 pages, it is possible to select 25, 50, 75, 100, ... 275 pages). The selected pages and two next pages (25–26–27, 50–51–52, 75–76–77, ...) are analyzed, the level of the physical information complexity on every three pages is estimated on the scale considered above. As a result, for each of the three pages we give the marks $B_{ij}$ ($j = 1, 2, ...$), which are entered in a table similar to tab. 1. Average value $B_{ij}^{av}$ for $i$-th textbook is calculated by the formula $B_{ij}^{av} = (B_{i1} + B_{i2} + ... + B_{in}) / n$, where $n$ – the number of selections.

3 To calculate physical complexity of the $i$-th textbook according to the formula

$$F_i = 0.25 \frac{A_i - 1}{4} + 0.75 \frac{B_{ij}^{av} - 1}{3.25}.$$  

The coefficients are selected so that it were possible to correct a contribution of estimates $A_i$ and $B_{ij}$ in the general assessment of physical complexity.

<table>
<thead>
<tr>
<th>Numbers of pages</th>
<th>40-42</th>
<th>80-82</th>
<th>120-122</th>
<th>160-162</th>
<th>200-202</th>
<th>240-242</th>
<th>280-282</th>
<th>320-322</th>
<th>360-362</th>
<th>400-402</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical complexity of text</td>
<td>$A_j$</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Number of the formulas</td>
<td>$N_{ij}^{F}$</td>
<td>13</td>
<td>6</td>
<td>12</td>
<td>8</td>
<td>2</td>
<td>12</td>
<td>10</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Complexity of the formulas</td>
<td>$K_{ij}^{F}$</td>
<td>2</td>
<td>2</td>
<td>2,5</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1,5</td>
<td>2</td>
<td>1,3</td>
<td>1</td>
</tr>
<tr>
<td>Number of pictures with the mathematical information</td>
<td>$N_{ij}^{P}$</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Mathematical complexity of text</td>
<td>$D_{ij}$</td>
<td>26</td>
<td>18</td>
<td>36</td>
<td>10</td>
<td>4</td>
<td>26</td>
<td>15</td>
<td>12</td>
<td>9,2</td>
<td>5</td>
</tr>
</tbody>
</table>

The mathematical complexity of the textbook is characterized by complexity and variety of the mathematical models used for the description of the studied phenomena and the solution of physical problems. Indirectly it can be defined by counting the number of formulas (taking into account their complexity) and pictures in which the mathematical abstractions (vectors, power lines, graphs) are represented. Mathematical complexity of the textbook is defined as follows:

1. To analyze the mathematical formulas presented in $i$-th textbook, and estimate the general level of their complexity (showing the level of the pupil's knowledge who is capable to understand these formulas): $C_i = 1$ – only arithmetic operations are used; $C_i = 2$ – formulas with square roots and/or powering; $C_i = 3$ – formulas with trigonometrical functions; $C_i = 4$ – logarithms and limits are used; $C_i = 5$ – formulas with differentials, derivatives, integrals, complex numbers; $C_i = 6$ – in the formulas the operators containing derivatives are used (nabla, Poisson's brackets, etc.).
Table 2. Didactic complexity assessment of various physics textbooks

<table>
<thead>
<tr>
<th>i</th>
<th>The name of the physics textbook</th>
<th>Physical complexity</th>
<th>Mathematical complexity</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Physics–7 (Peryshkin A.V., 1999)</td>
<td>1 1.40 0.09 1 0.40 0.45 0.02 0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Physics–8 (Peryshkin A.V., 2000)</td>
<td>2 1.85 0.26 1 1.90 3.80 0.10 0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Physics–9 (Peryshkin A.V., Gutnik E.M., 2003)</td>
<td>4 2.87 0.57 3 3.00 6.09 0.20 0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Physics–10 (Myakishev G.Ya., Bukhtovtsev B.B., Sotsky N.N., 2004)</td>
<td>4 2.09 0.44 3 12.63 20.20 0.62 0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Mechanics–10 (Balashov M.M., Gomonova A.I., etc., 2002)</td>
<td>1 1.42 0.10 4 7.83 22.13 0.57 0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Physics–10: Molecular physics. Thermodynamics. (Myakishev G. Ya., A.Z. Sinyakov, 2002)</td>
<td>3 2.86 0.55 3 8.00 18.00 0.48 0.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Physics–10: Physics. Electrodynamics (Myakishev G. Ya., Sinyakov A.Z., etc., 2002)</td>
<td>3 2.88 0.51 5 8.82 21.50 0.60 0.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Physics–11: Oscillations and waves (Myakishev G. I. A. Z. Sinyakov, 2010)</td>
<td>3 2.95 0.58 5 10.10 33.80 0.61 0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Physics–11: Optics. Quantum physics (Myakishev G. I., Sinyakov A.Z., 2013)</td>
<td>5 3.50 0.83 3 3.45 10.50 0.28 0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Physics–11 (Myakishev G. I., Bukhtovtsev B. B., Charugin V. M., 2000)</td>
<td>5 3.22 0.76 5 1.58 4.50 0.20 0.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Physics–10 (Kasyanov V.A., 2003)</td>
<td>4 2.50 0.53 4 7.50 15.48 0.46 0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Physics–11 (Kasyanov V.A., 2004)</td>
<td>5 3.30 0.78 5 7.70 16.12 0.50 0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Physics (Pinsky A.A., Grakovskiy G. Yu., 2008)</td>
<td>5 2.96 0.70 4 11.10 26.73 0.70 0.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Course of physics. T.1. Mechanics, molecular physics (Savel’yev I.V., 1989)</td>
<td>4 2.54 0.54 5 10.08 41.10 0.91 0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Course of physics. T.2. Electricity and magnetism. Waves. Optics. (Savel’yev I.V., 1988)</td>
<td>4 2.92 0.63 6 9.92 45.89 1.00 0.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Course of physics. T.3. Quantum optics. Nuclear physics. (Savel’yev I.V., 1987)</td>
<td>5 4.25 1.00 6 10.00 30.50 0.78 0.90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. To select \( n = 10–15 \) pages of the textbook which are uniformly distributed throughout the text. The selected pages and two following (altogether 3 pages) are analyzed; the number of formulas \( N_{ij}^F \), considering their complexity \( K_{ij}^F \), and the number of pictures \( N_{ij}^P \) that contain mathematical information (vectors, graphics, frame of reference) are counted. One picture containing mathematical abstractions is equated to a formula with complexity 2. To calculate the mathematical complexity of each of the three pages; for this purpose the number of formulas is multiplied by their complexity, and the result is added to the number of pictures multiplied by the weighting factor 2: \( D_{ij} = N_{ij}^F K_{ij}^F + 2N_{ij}^P \).

After that for every \( i \)-th textbook the average value \( D_{i}^{av} \) and average number of formulas on three pages are calculated as:

\[
D_{i}^{av} = \left( D_{i1} + D_{i2} + \ldots + D_{in} \right) / n, \quad N_{i}^{av} = \left( N_{i1}^{F} + N_{i2}^{F} + \ldots + N_{in}^{F} \right) / n.
\]

3. To find the complex indicator of mathematical complexity of the textbook

\[
M_i = \frac{1}{4.09} \left( 0.5C_i - \frac{N_i^F}{5} + \frac{N_i^F}{11} + 2D_i^{av} \right)
\]

The weight coefficients allow to correct the contribution of these estimates to the general assessment of mathematical complexity which should fill the interval \([0; 1] \).
4. To define the general complexity of the textbook \( S_i = (F_i^2 / 2 + M_i^2 / 2)^{0.5} \).


![Fig. 1. The distribution of textbooks in the feature space "physical complexity \( F \) – mathematical complexity \( M \)."
](image)

The calculation results of the physical complexity \( F \), mathematical complexity \( M \) and didactic complexity \( S \) of the physics textbooks are presented in tab. 2. In fig. 1 distribution of textbooks in the space formed by axes \( F \) and \( M \) is represented. The numbers near points coincide with numbers of textbooks \( i \) in tab. 2. It is visible that the high school textbooks on electrodynamics, optics and quantum physics (15), (16) have the greatest complexity, and the textbooks on physics for the 7–th class (1) and for the 8–th class (2) have the smallest complexity. The last two textbooks (1), (2), physics textbook (3) for 9–th class and textbooks (9), (10) for the 11–th class have the mathematical complexity \( M \) less than 0.4. Textbooks (3), (9), (10) have the physical complexity of more than 0.4, while their mathematical complexity is sufficiently low (0.2 - 0.3). The textbook for the 10–th class on mechanics (5) has low physical complexity (0.10), but rather high mathematical complexity (0.57). In this textbook the mechanical phenomena, the majority of which can be observed in everyday life, are considered, but at the same time rather difficult mathematical models are used. Textbooks for schools (4), (6), (7), (8), (11), (12), (13), as well as textbooks for higher education institutions (14), (15), (16) have physical and mathematical difficulties more than 0.4.

The considered above method allows "to measure" complexities \( F \) and \( M \) of various physics textbooks quickly enough and to make a judgement what textbooks it is appropriate to use in this or that situation. At the same time the content of educational texts, correctness of logical conclusions,
methodical validity of reasonings are not considered as it is initially assumed that the analyzed textbook corresponds to all standard requirements for this type of publications.

The offered technique of an assessment of physical and mathematical complexity allows to make the comparative analysis of various education texts. The results of this expertise can be taken into account while writing textbooks of new generation, and also in the work of teachers. It is well-known that pupils differ in their interests, knowledge of mathematics, and have unequal abilities to assimilate different types of information. The physical and mathematical complexity of the textbook can predict which students will acquire this or that education material better.

4 THE COMPLEXITY ESTIMATION OF TEXTBOOKS ON NATURAL SCIENTIFIC SUBJECTS

Learning the basics of natural sciences assumes the creation and analysis of the qualitative and quantitative models of the considered natural phenomena, the execution of practical jobs, the solution of tasks, etc. Therefore it is logical to assume that didactic complexity of the problems studied at school is determined by: 1) the variety and abstractness of qualitative explanations and their isolation from everyday life (characteristic $A$); 2) the complexity of mathematical methods and models, variety of considered quantitative tasks (characteristic $B$).

The complexity or abstractness of qualitative reasonings $A$ of the educational text is proportional to the degree of isolation of the studied issues from the everyday pupil's life. In the theory of knowledge the abstract object is opposed to the concrete thing [14]. The concrete thing perceived by sense organs ("this plant", "this ampermetre", "concrete tube with reagent") belongs to the lowest level of abstraction. Higher level of abstraction is the concept of generic essence of a thing ("hydrochloric acid in general", "any insect", "each tree"). The following level of abstractness corresponds to using the idealized models in the reasonings ("drop model of a nucleus", "electronic orbital", "model of the protein molecule") or objects ("photon", "positron", "chromosome") which the pupil can't observe in everyday life or in physical laboratory.

To estimate the complexity of qualitative reasonings $A$, it is necessary to determine the abstractness level of the used models, the degree of their isolation from the pupil's daily experience, and existence of a contradiction between them. We must take into account: 1) the possibility of perception of the studied object (or its analog) with the help of sense organs; 2) the change of object with time; 3) the number of the freedom degrees, the quantity of independent variables that determine the state of the object; 4) the sizes, spatial localization of an object or a temporary extension of the process; 5) the existence of structure and its complexity; 6) the correspondence of the object behavior to "common sense".

It is known that the natural study and geography in 4 – 6 classes include relatively simple objects, many of which (or similar) are found in every person's life (rivers, mountains, different plants, animals, insects). When considering some imaginary objects (the Earth's core, plant cell) the depth of theoretical study is not large. At the same time the learning of physics and chemistry courses in 9–11th classes requires the pupils’ developed abstract thinking. Even the consideration of mechanical and thermal phenomena supposes the use of the idealized models (material point, the molecule, the ideal gas) and a variety of the mathematical abstractions (frame of reference, vector and their projections, graphs, etc.). In the study of the bases of electrodynamics, optics, atomic and nuclear physics students are forced to imagine various objects (electromagnetic waves, atoms, elementary particles) and phenomena (the photoelectric effect, nuclear decay), which are not perceived by their senses directly, and some of them can not be studied experimentally using devices available in the physics classroom.

Fundamentals of natural sciences are reflected in the following school disciplines: the world around (3 – 4 classes), natural study (5 classes), geography (6 – 8 classes), biology (6 – 11 classes), ecology (10 – 11 classes), physics (7 – 11 classes) and chemistry (8 – 11 classes). For an estimation of characteristic $A$ (abstractness of qualitative explanations) of school textbooks, their analysis has been carried out for the assessment of complexity and a variety of the considered objects and processes. The expert has checked the textbook, paying attention to the names of themes and paragraphs, pictures, definitions and formulations of laws, finding the most difficult issues. The textbooks have been compared in pairs with each other, the result of the comparison is expressed in a notation of the mark $-1$, 0 or 1 in the corresponding cell of the table Excel. The mark 1 (or $-1$) on crossing the $i$–th line and the $j$–th column means that the amount of the estimated quality $A$ in the $i$–th textbook is
much more (or less), than in the \( j \)-th textbook. If the amounts of quality \( A \) in both textbooks are approximately equal, then they are marked as 0. After filling the square matrix, value \( A_i' \) for \( i \)-th the textbook has been calculated; for this purpose the sum of marks of the \( i \)-th column was subtracted from the sum of values of the \( i \)-th lines.

Fig. 2. The results of the comparison of textbooks under the characteristics \( A \) and \( B \).

The assessment of didactic complexity of the used mathematical models (the characteristic \( B \)) has been made in a similar way. Mathematical complexity of the educational text depends on: 1) the complexity and a variety of formulas and pictures containing mathematical abstractions; 2) the complexity and variety of the solved tasks; 3) the level of the mathematics knowledge demanded from the pupil for understanding of the material. In this sense the 9th classes physics textbook in which only arithmetic operations are used is much simpler than the 11th class physics textbook for understanding of which it is necessary to know trigonometrical formulas and derivatives. The estimation results received by the method of pair comparisons are normalized so that the turning -out \( A \) and \( B \) values lay in the range from 0 to 1 (fig. 2.1). The correlation coefficient between \( A \) and \( B \), calculated in the Excel program, is equal 0.51; it means that the chosen characteristics can be considered as independent.

The table in fig. 2.1 contains: 1) the list of the textbooks arranged in order of increasing didactic complexity \( S \); 2) the textbook code used in fig. 2.2; 3) the corresponding values of characteristics \( A \), \( B \) and didactic complexity \( S \) of the textbook. Fig. 2.2 shows the distribution of textbooks in the two-dimensional feature space formed by the mutually perpendicular axes \( A \) and \( B \). In this case, the didactic complexity of the \( i \)-th textbook proportional to the length of the segment connecting the \( i \)-th point and the beginning of coordinates O. The degree of difference between the two textbooks is determined by the distance between the points corresponding to them in this space of signs \( A \) and \( B \). For example, the geography textbook for the 8th class in this sense is closer to the biology textbook for the 8th class (distance 0.14), than to the 8th class chemistry textbook (distance 0.50).

From fig. 2.2 it is visible that all textbooks can be divided into the following groups: 1) textbooks of geography, natural study, ecology and biology for 6–8 class, having low complexity (DC less than 0.4); 2) the biology textbooks for the 9–11 classes with high complexity of qualitative reasonings \( A \) and low complexity of mathematical models \( B \); 3) textbooks of physics and chemistry for the 9–11 classes having high measure values \( A \) and \( B \) (S more than 0.65). Textbooks for various classes corresponding to one subject are connected with lines. It is visible that upon transition to the senior classes: 1) DC of biology textbooks increases due to increase in \( A \); 2) DC of the physics and chemistry textbooks increases due to growing of both characteristics \( A \) and \( B \); 3) DC of natural study, geography and ecology textbooks remains comparatively low. Physics textbooks exceed chemistry textbooks for the same classes on characteristic \( B \), but “lose” on \( A \).
geometry textbooks have high complexity of mathematical models, but almost do not contain qualitative models of the natural phenomena; so they fall into shaded area of space \( A - B \) (fig. 2.2).

5 CONCLUSIONS

In this article the following problems are solved: 1) the uniform criteria for estimation of the didactical complexity of the scientific concepts on various natural subjects are offered; 2) the method of determination of the texts didactical complexity which consists in counting of scientific terms number and considering of their abstractness degree by means of the computer is discussed; 3) the didactic complexity of 16 Russian physics textbooks for school and universities is assessed, their distribution in space of signs "physical complexity – mathematical complexity" is studied; 4) paired comparison of the Russian textbooks on natural study, geography, biology, physics, chemistry (21 textbooks) is made, their didactic complexity is estimated. The offered methods and results of an assessment of didactic complexity of educational texts can be used for comparison of various paragraphs, themes, subjects and detection of the corresponding regularities of the education information distribution.

REFERENCES


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