ABOUT THE USING OF THE COMPUTER MODELS FOR STUDYING OF THE DIDACTIC SYSTEMS

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Abstract

The main objective of the mathematical theory of training is as follows: on the basis of the pupil's characteristics, his initial knowledge level and distribution of the studied material, it is necessary to define quantity of the pupil's knowledge during training and after its ending. The aim of the work is the research of various mathematical models of training with using of the computer and creation of imitating (computer) models of the didactic system. Methodological basis of the research are works by N. Winer, K. Shannon, V. M. Glushkov, D. A. Pospelov (cybernetics, the theory of information), R. Atkinson, L.P. Leontyev, F.S. Roberts, L.B. Itelson (mathematical modelling of training), B. Skinner, S.I. Arkhangelsky, V.P. Bespalko, E. I. Mashbits, I.V. Robert (cybernetic approach in pedagogy, the automated training systems).

We use information–cybernetic approach in the analysis of the didactic system "teacher–pupil" on the basis of which three models are offered. The first model is based on the assumption that information given by the teacher consists of the blocks uniting 4–6 elements of a learning material (ELM). The pupil understands the new block of the given information only when he manages to comprehend each entering ELM before arrival of the next information block. For determination of the average speed of assimilation the method of statistical tests is used. The computer program simulates transferring of 2000 blocks and counts the number of the blocks comprehended by the pupil in the given interval of time at various speed of the information reporting by the teacher.

The second model considers that different ELM–s have unequal complexity and are remembered with various durability (strength). That knowledge which is included in the pupil's educational activity and which is demanded by him, is remembered much better and forgotten more slowly than the knowledge which the pupil doesn't use. At increase in number of the pupil's addressing to this ELM: 1) the time of his using it decreases, tending to some limit; 2) the forgetting coefficient decreases, tending to zero. This model allows to simulate the training at multiple addressing to a set of ELM–s during one or several lessons.

The third model is two-component model of training which considers that: 1) the state of the pupil in each timepoint is defined by the quantity of the weak knowledge and strong knowledge (abilities, skills);

2) weak knowledge is forgotten quicker than strong knowledge; 3) while training the quantity of the pupil's weak knowledge increases, and a piece of weak knowledge turn into stronger (more durable) knowledge;

4) after the end of training forgetting takes place: strong knowledge gradually turns into less strong, and the quantity of the weak knowledge decreases according to the exponential law.

This model also takes into account: 1) the distribution of educational information during the whole time of training at school; 2) the pupil's learning and forgetting coefficients in various years of training; 3) shares of educational information of the previous levels (forms) which are used by the pupil when studying new material, and also using them during vacation and after the training completion.

The mathematical equations modeling training are presented in the article and the graphs of dependence of the pupil's knowledge different types on the time which are turning out as the result of their solution on the computer are analysed.

Keywords: computer modeling, didactics, education, mathematical methods, pedagogy, pupil, simulations, teacher, training.

1 INTRODUCTION

The main task of the mathematical learning theory can be formulated in this way: on the basis of the student's characteristics, his initial level of knowledge and the distribution of the studied material, it is necessary to determine the quantity of student knowledge during and after the training [1]. Its solution can be found by means of the method of imitating modeling [2]. The essence of the imitating modeling method consists in creation of the computer program simulating behavior of the system "teacher–pupil" and carrying out a series of the numerical experiments to understand the system behavior and to assess various management strategies for its effective functioning. It helps to investigate process of training in the cases when carrying out a pedagogical experiment is impossible or impractical [3]. Such computer models supplements the qualitative reasoning, makes it more objective and validated; they can be used when carrying out pedagogical experiments assume big costs or lead to negative results. Varying the distribution of a training material, lessons duration and pupil's parameters, it is possible to research influence of these factors on the process of the new knowledge assimilation and to find an optimum way of training in the given conditions. In this case scientists use mathematical models [4, 5, 6], discrete and continuous models, based on the automatic approach [7] and solution of differential equations [1, 3, 8], multiagent models [9], Petri nets, genetic algorithms, matrix models, etc.

The main objective of this work consists in research of various mathematical models of training on the computer and creation of simulation models of didactic system. Methodological basis of the research are ideas and works by N. Winer, K. Shannon [2], F. Rosenblatt, V.M. Glushkov, D.A. Pospelov (cybernetics, the theory of information), R. Atkinson, O.G. Gohman, L.P. Leontyev [4], F.S. Roberts [5], L.B. Itelson (mathematical modelling of training), B. Skinner, N. Krauder, S.I. Arkhangelsky, V.P. Bespalko, E.I. Mashbits, V.E. Firstov [10], V.S. Avanesov, I.V. Robert (cybernetic approach in pedagogy, the programmed training and the automated training systems). The computer modeling method is advantageously distinguished from "the method of qualitative reasonings or considerations" by logicality and formalization, reproducibility and concreteness. But there is a problem of correct choice of the model parameters. R. Shannon notes that almost any parameters of the simulation model often determines only on the basis of the experts assumptions, analyzing a small amount of data [2]. In our case the parameters of models are selected so that the turning-out results correspond training of the pupil who fulfills requirements of the training program successfully (for 70–80%).

2 DEPENDENCE OF ASSIMILATION ON THE SPEED OF THE INFORMATION REPORTING (MODEL 1)

Let us assume that information given by the teacher consists of separate blocks which are divided into the elements of a learning material (ELM-s): concepts, formulas, foreign words, pictures, text blocks, etc. Training is similar to watching a film when on the screen the information blocks (a text, drawings, formulas) come one after another. The pupil understands a new block of the given information if he/she understands each ELM entering this block. If the pupil hasn't acquired any ELM in the block, then he/she doesn't acquire the block in general (or can not solve a problem, prove a theorem, translate a sentence). Let in the block be m ELM-s, then assimilation time of one block by the pupil can't exceed $t_1 = m/u$, where u – the speed of reporting of the teacher's information (number of ELM-s for one conventional unit of time or CUT). When the pupil understands all ELM-s, he understands the entire block. The probability of understanding one ELM from the first attempt is equal $p =$ 0,7–0,9. If the pupil has not understood ELM from the first attempt, then he/she turns to it again and again until he/she understands, or until time t_1 meant for learning this block is be finished. Let all ELM-s have the information volume $I_1 = 1$. The time of the single reference of the pupil to every ELM (or time of operation) is $t_{op} = 1/\upsilon_M$, where υ_M – the speed of his mental activity in CUT $^{-1}$. The more cogitative actions for 1 CUT the pupil makes, the quicker he/she considers and understands this ELM. On the basis of these reasonings a computer program on Pascal was created, which allows to study the dependence of the acquired knowledge quantity on the arrival rate of the teacher's information reporting (fig. 1.1 and 1.2).

The program displays the number of the reported information blocks, the number of the comprehended blocks and the expended time that allows to determine the reporting speed and the knowledge assimilation speed (in block/ELM). In fig. 1.1 the graphs of dependence of the assimilation speed V on the speed u of the teacher's information reporting (in ELM/CUT) at various values of the pupil's thinking speed υ_M are presented. If the speed u of the teacher's information reporting is small then the pupil's assimilation speed is equal $V = dI_p / dt = u' = u/5$. The pupil with this speed of mental operations υ_M manages to understand almost all blocks which are reported by the teacher. Further increase in the speed of information reporting u leads to the fact that the pupil does not have time to accept it completely: the assimilation rate V reaches a maximum and then decreases. The higher the speed of the pupil's mental operations υ_M, the greater the maximum possible speed of assimilation V and rate u of the teacher's information reporting corresponding to it. In the book [4, pp. 108–157] various mathematical models $I_p = \varphi(I)$ connecting volume I_p of the material acquired by the pupil and the information I given at the lecture are considered. Presented in fig. 1.1 curves are similar to the graphic of the function $I_p = \varphi(I)$ from [4]. This is because $I = u'T = uT/5$, $I_p = VT$, where T is the lecture duration. The coefficient of material understanding is equal to the relation of number of the comprehended blocks to the total information reported by the teacher: $K = I_p / I$. At low speeds of information reporting u the pupil understands practically everything what is reported by the teacher, therefore $K = 1$. In the process of increasing u the understanding coefficient K gradually decreases to 0 (fig. 1.2). While increasing the speed of mental operations υ_M, the speed of the teacher's information reporting u, which corresponds to $K = 0{,}5$, grows.

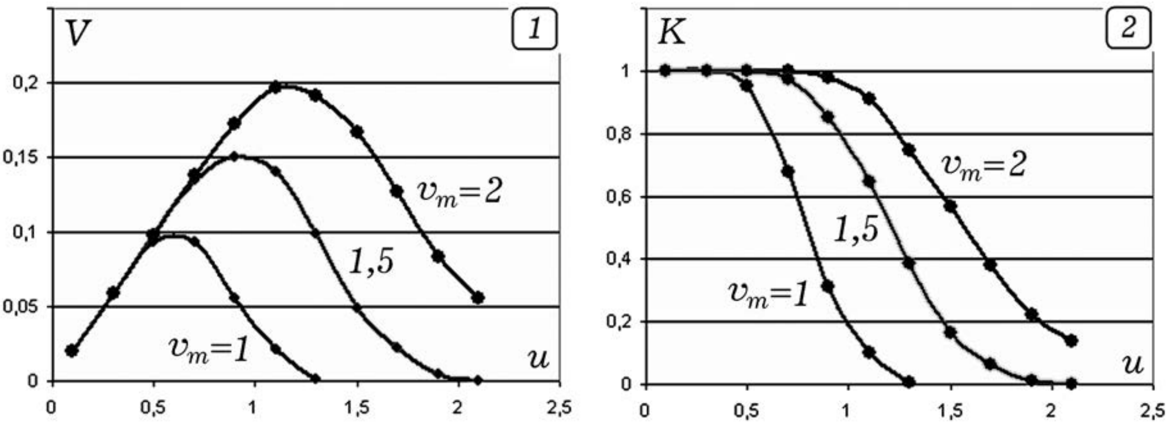

Fig. 1. Dependence of assimilation on the speed of the information reporting.

Using the spreadsheets Excel, it has become possible to select the function corresponding to the received graphs (fig. 1.1). For $\upsilon_M = 1{,}5$:

$$V(u') = \frac{dI_p}{dt} = \frac{u'}{1 + \exp(a(u' - b))}, \quad u' = \frac{u}{5}, \quad a = 27 \text{ УЕВ}, \quad b = 0{,}24 \text{ 1/УЕВ}.$$

Here u' and V show the quantity of the reported and acquired blocks for one CUT. If $u < 0{,}7$, then $1 + \exp(a(u/5 - b)) \approx 1$, the rate of increase of the pupil's knowledge (block/CUT) is equal to the speed of the teacher's knowledge reporting $u' = u/5$. If $u > 1$, then V decreases, approaching to 0. In the training process the pupil's speed of thinking υ_M increases, the ability to acquire new material grows. The following values of parameters correspond speeds υ_M = 1, 1.5 and 2: a =39, b = 0,16; a =27, b = 0,24 and a =18, b = 0,32. If υ_M grows, then a decreases, and b increases.

The resulting graphics can be interpreted in a different way. For example, the pupil solves the problems, each of which requires m complex operations. The probability of correct execution of each operation equals p. If the pupil does not fulfill the given operation from the first attempt, he/she makes another attempt, and so on, each time spending $t_{op} = 1/\upsilon_M$ CUT. For each task a fixed time

$t_1 = m/u$ is given. The graphs in fig. 1.2 show the dependence of the problem solving probability on the speed of their arrival $u' = u/5$ (inversely proportional t_1), with different speeds of thinking υ_M.

3 ACCOUNTING OF THE DEPENDENCE OF THE FORGETTING SPEED ON THE NUMBER OF REPETITION (MODEL 2)

It is often assumed that all elements of the educational material are assimilated equally firmly [6, 7, 9]. However, it is well-known that the knowledge that is included in the pupil's training activities, remembered much better than the knowledge that he does not use [1, 3, 8]. Let us take the pupil who, in the course of training, must solve a sequence of certain tasks within the same theme. For example, during a lesson he/she has to sum numbers (or to read separate words, to perform tasks of a test) in certain timepoints. The rest of the time at a lesson he/she is engaged in other educational activity which doesn't interest us. At the moment t_i let the pupil start solving a task in the i–th time, and while he/she is doing that the level of assimilation of the corresponding ELM increases to maximum value $Z = 1$. We consider that the time τ of the solution of the task (or time spent for the work with this ELM) depends on, how many times s this problem was solved earlier. It is possible to suppose that with growth s the time τ decreases according the following law: $\tau = 1 + 1{,}5e^{-s/5}$ CUT, aspiring to $\tau_\infty =$ 1 CUT. Having performed the task and increased the level of knowledge of the corresponding ELM to 1, the pupil turns to the solution of another educational task and starts forgetting the given ELM according to the law of forgetting $dZ/dt = -\gamma \cdot Z$. If the number s of using this ELM grows, then the given ELM is remembered better. The forgetting coefficient for this ELM decreases, for example, according to such law: $\gamma = 0{,}002e^{-s}$ (CUT^{-1}). We use the computer Pascal program which models training when addressing to one ELM in timepoints 3, 6, 9, 12, 15, 18 CUT. The results of modeling are presented in fig. 2.1. It is visible that after the first and second turning to the given ELM the acquired knowledge is forgotten quickly, and after the fifth and the sixth – it is forgotten very slowly. As a result of repeated use of this ELM the coefficient of forgetting decreases practically to 0, the information is remembered well.

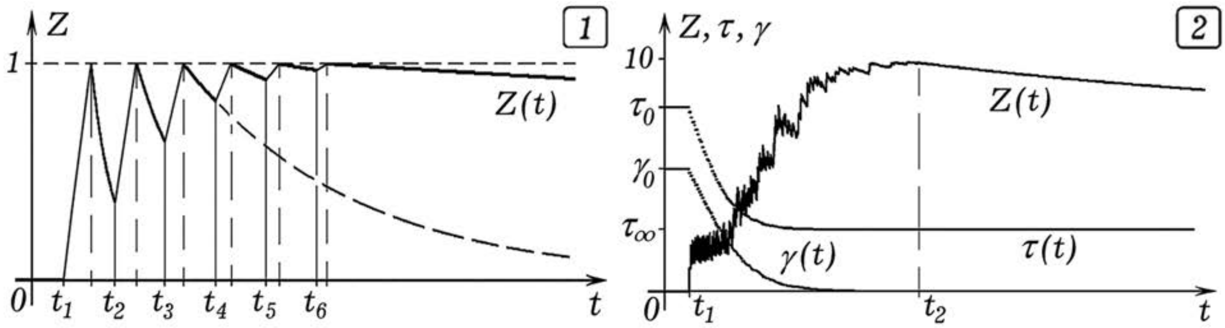

Fig. 2. Results of modeling: 1) change of the knowledge level of one ELM as a result of 6 repetitions; 2) studying 10 ELM-s during $T = 300$ CUT.

Now let us create the model of studying N ELM-s in the duration T of the lesson. For example, the pupil studies N new words of a foreign language which are numbered from 0 to N. Reading the text, at the moment t_1 the pupil meets word 2 and during time τ_2 translates it, at the moment t_2 he/she meets word 5 and during time τ_5 translates it, at the moment t_3 – he/she meets word 1, etc. When the pupil translates the i–th word for the first time ($s_i = 1$), he addresses to the dictionary and writes out the word meaning, the second time – he looks it up in his notebook, the third time – he translates from his memory, etc., each time he spends less time τ_i. We consider that these ELM-s are met by the pupil in a random way and for work with the i–th ELM he spends $\tau_i = 1 + 2e^{-s_i/2}$ CUT (the time for the solution of the i–th task), where s_i – is the number of appealing. In process of increasing s_i

there is a reduction of forgetting coefficient for the i-th ELM according to the law $\gamma_i = 0{,}002 e^{-s_i/3}$ CUT $^{-1}$.

The used program builds graphs of the dependences: 1) of the total knowledge quantity Z on the time; 2) of the average address time τ for all ELM-s on the time; 3) of the average forgetting coefficient γ for all ELM-s on the time. The resulted curves at $N=10$ and $T=300$ CUT are represented in fig. 2.2. It is visible that on average while training the total level of knowledge raises, the average time τ of the task solution decreases, aspiring to the limit τ_∞, the average forgetting coefficient γ decreases, tending to zero.

Results of modeling for $N=6$ and 10 at identical durations of a lesson are presented in fig. 3.1. If the number of the studied ELM-s N increases, the number s_i of turning to each ELM decreases therefore they are remembered worse. As a result, at the end of training the average forgetting coefficient γ is too big. Therefore, at the end of training for $N=10$ the quantity of knowledge decreases owing to forgetting. At $N=6$ the quantity of knowledge remains almost constant. If number of the studied ELM-s grows then at the end of training the curve $Z(t)$ decreases quickly. The result of modeling at $N=20$ is given in fig. 3.2. So, the analyzed model shows that the number of ELM-s N studied at one lesson shouldn't be too great. At big N the pupil acquires knowledge worse and then it is quickly forgotten.

And now let us study the dependence of the pupil's knowledge quantity Z from number N of the studied ELM-s (which is proportional to the reporting speed of the educational information $\upsilon = N/T$). We change the computer model so that the studied ELM-s would follow not in a random way but one after another, and we calculate the level of the pupil's knowledge Z and the average coefficient of forgetting γ at the $N=3$. After that we repeat calculations at $N=5$, 8, 11, …, 20. In our case the duration of the lesson was $T=300$ CUT, and control time is $t'=350$ CUT. Also we calculate the indicator of training efficiency $K=Z/N$ which is equal to the relation of the knowledge quantity Z at the moment $T+t'$ to total number N of the studied ELM-s.

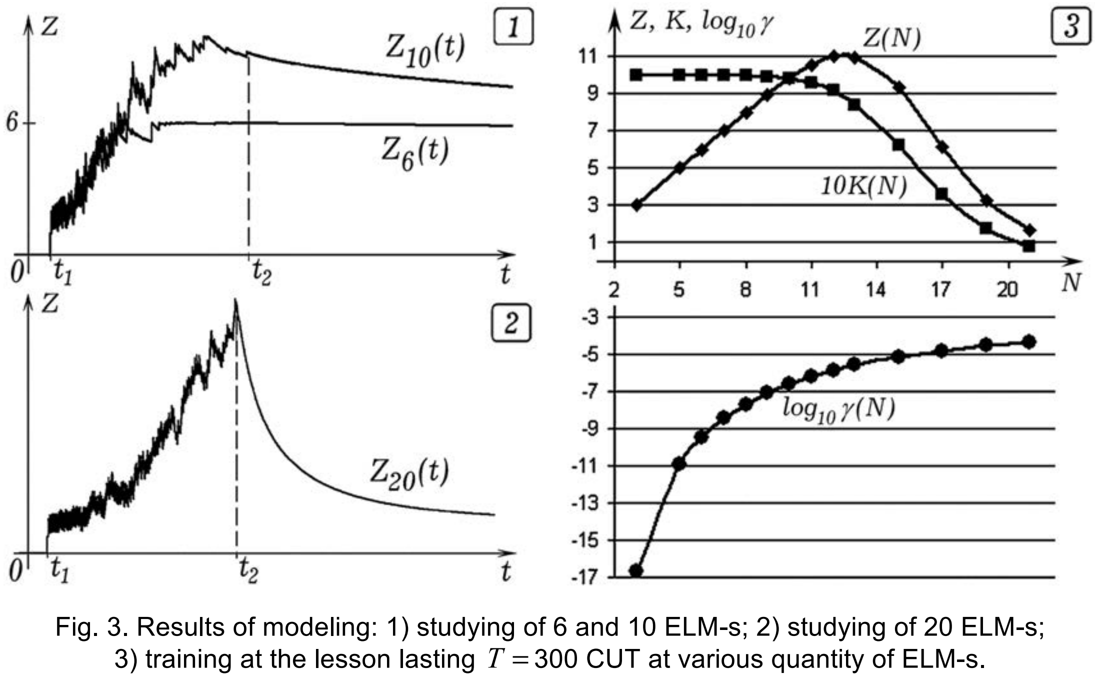

Fig. 3. Results of modeling: 1) studying of 6 and 10 ELM-s; 2) studying of 20 ELM-s; 3) training at the lesson lasting $T=300$ CUT at various quantity of ELM-s.

Results of modeling are given in fig. 3.3. It is visible that the graphs $Z(N)$ and $K(N)$ look like the curves in fig. 1. With growth N from 3 to 21 the average coefficient of forgetting γ grows from $2\cdot 10^{-17}$ to $4\cdot 10^{-5}$, so at the average, ELM-s are acquired worse and forgotten quicker. With growth

N the pupil's knowledge Z in time t' after the end of training increases, reaches a maximum at $N = 12$, and then at $N > 12$ decreases. It is explained by the influence of two factors: 1) the increasing of the studied ELM-s number N; 2) the reduction of the number of appeals s_i to each ELM during time T and, as a result, deterioration of the knowledge assimilation (increasing of the forgetting coefficient γ). At small N the indicator of training efficiency K is equal 1, and with growth N it decreases to zero. So, there is such N, at which in time t' after the end of training the pupil's knowledge quantity Z is maximum. The lesson's duration is constant and equal T. If the rate of the educational information reporting $\upsilon = N/T$ increases then the pupil's knowledge quantity at the moment t', first increases, reaches the maximum and then decreases.

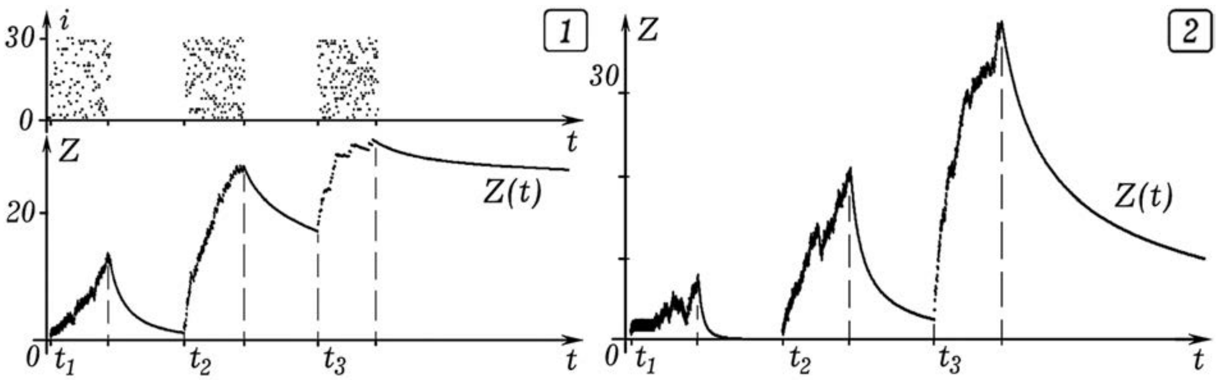

Fig. 4. Simulation results: the study 30 and 60 ELM-s on three lessons.

Now let us create the model of studying $N = 30$ ELM-s during three lessons lasting $T = 180$ CUT. The lessons are divided by breaks lasting $T_n = 220$ CUT. At the lessons the pupil addresses to one or to another ELM with equal probabilities. When the appealing number s_i for the i–th ELM grows, the spent time τ_i and the forgetting coefficient γ_i decrease. The results of imitating modeling are given in fig. 4.1. It is visible that while training the number i of the considered ELM changes incidentally from 1 to 30, and the level of the pupil's knowledge Z increases. During breaks there is forgetting, and Z decreases. In fig. 4.2 the results of imitating modeling of studying 60 ELM are presented. It is visible that the level of knowledge after training is much higher, than in fig. 4.1. The pupil addresses to each ELM smaller number of times, therefore the studied information is forgotten quicker.

4 TWO–COMPONENT MODEL OF THE DIDACTIC SYSTEM (MODEL 3)

An alternative approach is possible which allows to consider that the knowledge which is included in the pupil's educational activity, is remembered better, than the knowledge which he doesn't use. At the same time it is supposed that: 1) the durability of assimilation of various ELM isn't identical, therefore they should be divided into several categories; 2) the strong (durable) knowledge is forgotten significantly slower than week (poor) knowledge; 3) while training the pupil's weak knowledge gradually becomes strong and turns into skills. This approach has allowed to construct a four-, three- and two-component models of learning which are analysed in [8, p. 69 – 72].

Let us consider two-component model of training at 11-year school which is expressed by equations:

$$dZ_i/dt = \alpha_i(U_i - Zn_i) - \beta_i Z_i - \gamma_{Zi} Z_i, \quad dN_i/dt = \beta_i Z_i - \gamma_{Ni} N_i, \quad i = 1, 2, \ldots, 11.$$

Here U_i – the level of the teacher's requirements in the i–th class, equal to the amount of knowledge Z_{0i} reported by him, $Zn_i = Z_i + N_i$ – the total pupil's knowledge for the i–th class, Z_i – the quantity of weak knowledge for the i–th class, which has high forgetting coefficient γ_{Zi}, and N_i – quantity of the pupil's strong knowledge for the i–th class which has low forgetting coefficient γ_{Ni}. At any moment t the pupil's state is defined by matrixes $Z_i = (Z_1, Z_2, \ldots, Z_{11})$ and $N_i = (N_1, N_2, \ldots, N_{11})$, characterizing amounts of the acquired strong and weak knowledge which are included into the training program for 1st, 2nd, …, 11th classes. When training in the i–th class the values Z_i и N_i ($i=1$, 2, … 11) grow, the knowledge amounts for the previous classes which the pupil addresses increases also. The total pupil's knowledge for the i–th, the $(i+1)$–th, …, the j–th classes is equal $Zn_{i-j} = Zn_i + Zn_{i+1} + ... + Zn_j$. The pupil's assimilation coefficient of a training material for the i–th class is $K_i(t) = Zn_i(t)/U_i = (Z_i(t)+N_i(t))/U_i$. The coefficient of the pupil's knowledge durability is equal to a share of strong knowledge from total amounts of the pupil's knowledge at the given time t: $P(t) = N(t)/(Z(t)+N(t)) = N(t)/Zn(t)$.

The considered model of training should take into account: 1) the main regularities of training and forgetting [11]; 2) the transformation of the weak pupil's knowledge into strong knowledge (or skills) for which the forgetting coefficient is smaller; 3) the increase of the studied information quantity and its complexity (abstractness degree) at the transition of the pupil to the senior classes; 4) the increase of the pupil's assimilation coefficient at the transition to the next class; 5) the using by the j–th class pupil of the training material studied in previous 1st, 2nd, …, $(j-1)$–th classes; 6) the application of knowledge, represented in the textbook for the j– th class, in everyday life during holidays and after graduating school. For modeling of pupil's knowledge dynamics while training at school and after its termination it is necessary to set the schoolchild's parameters, his/her initial condition at the moment t = 0 and the teacher's influence. The pupil is described by:

1 The assimilation coefficients α_i which characterizes the transition speed of the teacher's knowledge $Z_{0i} = U_i$ for the i–th class to the weak pupil's knowledge Z_i. The values α_i in the training process monotonously increase, since the more information a pupil has acquired, the easier he/she remembers new information. The coefficients α_i can be set as this: $a_i =$(1, 1.2, 1.4, 1.6, 1.9, 2.2, 2.5, 2.8, 3.1, 3.4, 3.7), $\alpha_i = a_i/12$.

2 The coefficients of formation of the strong knowledge $\beta_i = \alpha_i/80$ ($i=1$, 2, … 11) which characterizes the transformation speed of the weak knowledge into the strong knowledge; here Z_i decreases and N_i increases by the same value.

3 The forgetting coefficients of the weak and strong knowledge obtained in the i–th class. It is known that the knowledge gained in the 1st – 4th classes is used by the pupil in everyday life and therefore is remembered well. In senior classes the abstractness of educational information increases, so the acquired knowledge is more detached from everyday life and have higher rate of forgetting. These coefficients can be set as follows: $g_i =$(10, 10, 10, 11, 12, 13, 14, 15, 16, 17, 18), $\gamma_{Zi} = g_i/200$, $\gamma_{Ni} = \gamma_{Zi}/60$. As the forgetting occurs under the exponential law, the time of forgetting a half of the available knowledge is equal $T = \ln 2/\gamma$. For example, for the sixth class γ_{Z6}= 0,065 month^{-1} and γ_{N6} = 0,00108 month^{-1}. It means that for the weak knowledge which gained in the sixth class $T_Z \approx$10,7 months, and for strong pupil's knowledge $T_N \approx$53 years.

The following values characterize the external influence made on the pupil:

1 The distribution of educational information during the whole time of training at school; it is given by the array $U_i =$(10, 12, 14, 17, 20, 24, 29, 35, 42, 50, 59). Here U_i – the level of the teacher's requirements in the i–th class which is equal to the amount of the knowledge reported by him (in conventional units).

2 The reference coefficients of the j–th class pupil to the knowledge gained in the i–th class, set by a two-dimensional triangular matrix:

$$\varepsilon_{ij} = \begin{pmatrix} 1 & 0 & 0 & ... & 0 \\ 0,7 & 1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0,8 & 0,8 & 0,7 & ... & 1 \end{pmatrix}.$$

From $\varepsilon_{3,11}$= 0,7 it follows that, being trained in the eleventh class, the pupil uses 70 percent of the knowledge gained in the third class.

3 The coefficient of using of the information learned in the i–th class, during the holidays and after training. It can be set as follows: $c_i = 0{,}3$, if $i < 5$ and $c_i = 0{,}3/(i-4)$, if $i \geq 5$. In the mentioned intervals of time the pupil reads books, performs mathematical operations, watches films, speaks a foreign language, uses a variety of devices and software products. In result the knowledge received in the 1st – 4th classes increase and are assimilated more than the knowledge from the 9th – 11th classes with higher level of abstractness.

The values U_i, ε_{ij} and c_i should reflect the features of the school curriculum. Parameters α_i, β_i, γ_{Zi} and γ_{Ni} characterize the hypothetical pupil who studies successfully at school. In the used computer program time t is measured in months. It is considered that out of 12 months (one year) the pupil has a rest for 3 months, and for 9 months he/she studies at school. For the origin t = 0 the first day of training in the first class is taken, the initial level of the pupil's knowledge is equal $Zn(0) = 0$.

5 RESULTS OF MODELLING

In fig. 5 – 7 the imitating modeling results of the knowledge amount change for the hypothetical pupil within 11 years of school attendance and 10 years after graduation are shown. It is clear, that in practice various situations happen which differ in both the training curriculum, and parameters of specific pupils.

The graphs of the dependences of the pupil's knowledge amount for 1st – 4th, 1st – 8th and 1st – 11th classes on time are given in fig. 5.1. From fig. 5.2 it is visible that on average in the learning process (1st – 11th years) the total amount of knowledge $Zn_{1-11}(t)$ increases, and after graduating school – decreases, first of all, because of forgetting weak knowledge. The dips in graphics $Zn_{1-11}(t)$ correspond to summer holidays during which the school student forgets some weak knowledge. While training the amount of firmly acquired knowledge (or skills) $N_{1-11}(t)$ increases, and after the 11th class it practically does not change. The graphs $Zn_{1-4}(t)$ and $N_{1-4}(t)$ show the change in dynamics of the total knowledge amount and skills (strong knowledge) amount corresponding to the 1st – 4th classes during all considered interval t from 0 to 20 years. We mean the skills of reading, writing, arithmetic operations, basic knowledge about the world that a person learns in elementary school and then uses during his life. It is visible that the knowledge quantity monotonously increases, tending to the limit value $U_{1-4} = U_1 + U_2 + U_3 + U_4$, that equals the information which the pupil in the 1st – 4th classes should ideally acquire.

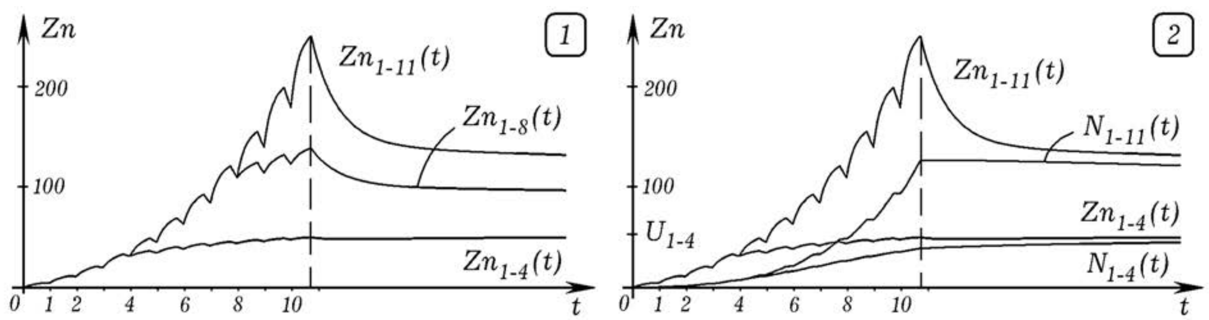

Fig. 5. The quantity of pupil's knowledge when training at school and after it graduation.

The graphs of change of knowledge amount $Zn_{5-8}(t)$ and $N_{5-8}(t)$ studied in the 5th – 8th classes are shown in fig. 6.1. The training material has higher level of abstractness and less used in everyday life, therefore: 1) the total knowledge amount Zn_{5-8} by the end of school (t = 11 years) is approximately equal 0,8U_{5-8}; 2) the amount of strong knowledge N_{5-8} is approximately equal 0,5U_{5-8}, where $U_{5-8} = \, = U_5 + U_6 + U_7 + U_8$ is the general level of the teacher's requirements in the 5th – 8th classes. As the knowledge acquired in the 5th – 8th classes is also partially used in everyday life, after the end of training their total quantity at first decreases, and then remains constant at the level of 0,5U_{5-8}.

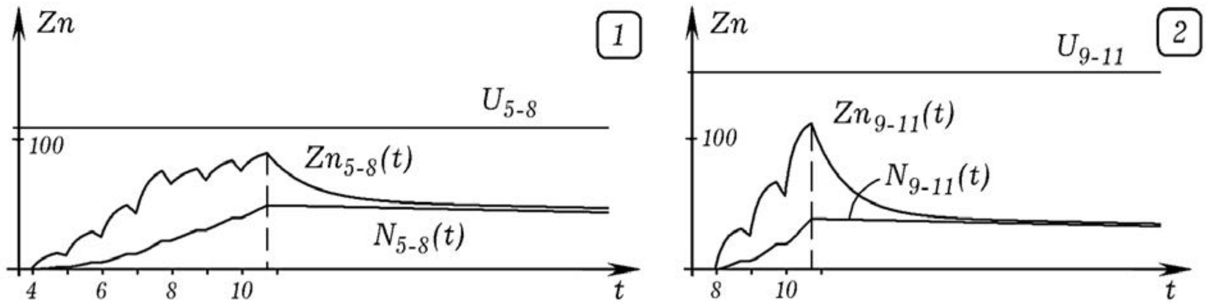

Fig. 6. Change of quantities of the knowledge studied in 5th – 8th and 9th – 11th classes.

The graphs $Zn_{9-11}(t)$ and $N_{9-11}(t)$ showing changes in the total knowledge amount and the strong knowledge amount studied in the 9th – 11th classes are presented in fig. 6.2. It follows that at the end of training (t =11 years) the total amount of knowledge reaches the maximum 0,7U_{9-11} while N_{9-11} is approximately equal to 0,25U_{9-11}. Here U_{9-11} is the total quantity of educational information presented in the textbooks for the 9th – 11th classes which the pupil should ideally acquire. While selecting model parameters it was considered that after successful training at school the hypothetical pupil remembers more than 0,6U_{9-11}, and during summer holidays after the 10th class he/she forgets about a third of the material acquired in the 10th class (fig. 6.2).

The offered model allows to study the time dependence of the pupil's assimilation coefficients $K_i^{zn}(i) = Zn_i / U_i$, $K_i^n(i) = N_i / U_i$ ($i = 1$, 2, … 11) on time or the number i of class. In fig. 7.1 shows the distribution $K_i^{zn}(i)$ and $K_i^n(i)$ at moment t = 8 years (the beginning of the 9th class); in fig. 7.2 – at the moment $t = 11$ years (3 months after graduating school); in fig. 7.3 – at the moment $t = 15$ years. It is visible that at the moments t = 8 years and 11 years the values $K_i^{zn}(i)$ и $K_i^n(i)$ almost monotonously decrease with the growth of number i (fig. 7.1 and 7.2). At the moment of t = 8 years K_i^{zn} =0,8–0,9, K_9^{zn} = $K_{10}^{zn} = K_{11}^{zn}$ =0, and at $t = 11$ years $K_1^{zn} \approx 1$, K_{11}^{zn} =0,7 – 0,8. After the graduation of school there is forgetting of weak knowledge, its quantity quickly decreases. The level of strong knowledge corresponding to the 1st – 4th class remains high because they are used in everyday life (fig. 7.3). For $i < 5$ the share of the acquired pupil's knowledge is high (K_i^{zn} =0,85 – 0,95), and after that in process of i increasing the coefficient K_i^{zn} smoothly decreases to 0,3.

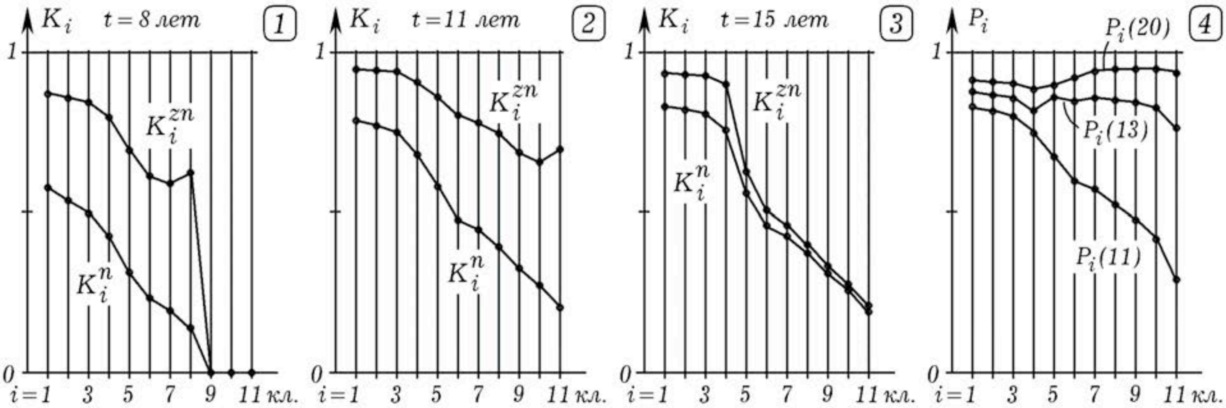

Fig. 7. The share of the acquired knowledge and their durability in dependence on the class number.

Fig. 7.4 shows the dependence of the acquired knowledge durability $P_i(t) = N_i(t) / Zn_i(t)$ on the number of a class i after the end of training at the moments $t = 11$, 13 and 20 years. It is visible that right after leaving school ($t = 11$ years) the knowledge of the 1th – 4th classes is the best ($P_i(11) = 0{,}6$ – 0,8), and durability of knowledge from the 9th – 11th classes lies in the range of 0,35 – 0,6. Eventually because of the forgetting the amount of weak knowledge decreases quicker, than strong (especially for $i > 4$), therefore the durability coefficient of a person's knowledge increases, and at time $t = 20$ years for all i it is approximately equal to 0,9. Improvement of model requires revision and clarification of the coefficients entering it.

So, the simulation model of training at school is offered which takes into account that: 1) the change of the knowledge amount takes place in accordance with the well-known laws of the person's learning and forgetting; 2) during training against the background of increase in the total number of pupil's knowledge, there is the transition of the weak knowledge into strong knowledge, which is forgotten slower; 3) with the growth of the class number the quantity of the studied information and its complexity (abstractness degree) increase, the forgotten coefficient grows; 4) in the process of learning at school there is an increase of the pupil's assimilation coefficient; 5) while training in the j–th class the pupil uses the material studied in the 1st, 2nd, … $(j-1)$–th classes; 6) during holidays and after graduation the person uses the studied material in everyday life, the part of weak knowledge becomes strong. After reasonable selection of the coefficients the offered model allows to calculate approximately the change of the pupil's knowledge amount and its qualitative structure over time.

6 CONCLUSION

The article is devoted to the research of the following computer models of training: 1) the probabilistic model considering that assimilation of information given by the teacher happens in small portions (blocks); 2) the model considering that because of increase in number of pupil's appeals to the given ELM, the time of its use and forgetting coefficient decreases; 3) the two–component model of training at eleven-year school considering transition of the weak knowledge into strong knowledge and the distribution of educational information between classes. These models allows to investigate and prove: 1) the dependence of assimilation of educational information on the teacher's reporting speed; 2) the existence of some optimum quantity of the educational information transmitted to the pupil during a lesson at which the level of the acquired knowledge is maximum; 3) the change of the assimilation strength (durability) and level of the knowledge which is studied by the pupil in the 1st, 2nd, …, 11th classes over time. With the help of computer models and by changing parameters of educational process and the pupil, we can find the best way of training in a concrete case [8, pp. 86 – 91].

The problem of creating the training computer program that simulates the educational process at school presents some interest. Such computer program can be used for training students of pedagogical institutions. It must allow to change the pupil's parameters, the duration of lessons, the distributions of a training material and strategy of the teacher's behavior. In the course of work with this program the student playing a role of the teacher changes the speed of the educational information reporting, quickly reacts to pupils' questions, holds examinations, gives marks, trying to achieve the highest level of knowledge for the given time. After the end of "training" the program draws the graphs showing change of “quantity of all pupils’ knowledge” and estimations for "the performed examinations". Besides, the training program can analyse the work of “the teacher” and give him a mark. The example of such program is the simSchool system [12].

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